Linkage without Interest for a Jewish State

English text with Hebrew translation.

Apart from section 8 and the appendix which are detailed, this article is a shortened and simplified version of the paper "Interest Free Wage Linkage of Personal Loans and Mortgages". Here we have omitted tables and references. The full paper can be found on my internet page - see end of article. Sincere thanks to colleagues, students, family, and friends for their help.

It is permitted to copy this article but only in its entirety.

I arrived in Israel in December 1978 - ספטמבר 1978, during a period of high inflation that continued for a number of years. There was economic turmoil.

During those years prices increased by 25% every month or two, and penalty interest rates reached about 1760% per annum. I clearly remember seeing a sign on a shop selling car accessories warning - "deferred payments bear interest of 4% a day".

During those years, three zeroes were removed from the currency - remember the lira and the shekel before the new shekel. Sadly, there was a state of economic collapse. Tragically, someone committed suicide because of heavy debts. It is a pity that all this happened amongst a people required by the Lord not to exact interest from each other.

In spite of all this, there were glimmerings of hope during those years of high inflation. The המודל של עליה, and borrowed but is this true for time? If time is money, does it have a representative exchange rate (ilation? If time is money, can it be used for defining prices, apartment rentals, fees, balances, monthly payments, rates and so on? By using the average wage for debt linkage, we can get quite close to lending and borrowing time. Yes, time is money and the average wage is its representative exchange rate. Yes, prices, apartment rentals, fees, balances, monthly payments, rates and so on can be defined in terms of time.

Lending with interest has generated hatred and persecution of the Jewish people and we must not forget

It is permitted to copy this article - see end of article. Since then, there was an economic turmoil.

Jonathan Tisdale

תורמ לגזרה, והרי חוסל בברך, ואני בברך מיכן ברוך, והרי חוסל בברך, ואני בברך מיכן ברוך. חוסל בברך, ואני בברך מיכן ברוך. חוסל בברך, ואני בברך מיכן ברוך.

It is permitted to copy this article but only in its entirety.
the lesson of history. Here I have tried to propose an alternative, which is fair to borrower and lender, and takes into account moral and economic considerations. My prayer and my plea, is that we remember the lesson of history, and make appropriate changes.

This concludes the opening remarks. Here are the contents of the article.

1 Index formulae and linkage
2 Accuracy and stability of index formulae
3 Wage and not price linkage
4 Linkage without interest
5 Accounting in terms of the average wage, discouraging fraud, encouraging honesty
6 Linkage and economic stability
7 Others who have proposed linkage without interest
8 Halachic questions
9 Conclusions and suggestions

Appendix - A layman's guide to index formulae

1 Index formulae and linkage

Let us say a family wishes to measure the change in the price of fruit and records details of the "fruit baskets" it purchases.

Initial purchase: 3 kilos apples at 3 coins a kilo and 5 kilos bananas at 2 coins a kilo.

Most recent purchase: 4 kilos apples at 2 coins a kilo and 1 kilo bananas at 4 coin a kilo.

Here are five values of the "family index of fruit" at the most recent purchase relative to the initial purchase, according to five different index formulae. (Detailed explanations and calculations appear in the appendix.)

<table>
<thead>
<tr>
<th>Index formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laspeyres</td>
<td>136.8</td>
</tr>
<tr>
<td>Paasche</td>
<td>85.7</td>
</tr>
<tr>
<td>Fisher</td>
<td>108.3</td>
</tr>
<tr>
<td>Unit</td>
<td>101.1</td>
</tr>
<tr>
<td>Normalized Unit</td>
<td>99.3</td>
</tr>
</tbody>
</table>

An index value of 100 is supposed to indicate that there is no price change. So it is not clear from the above, if there has been a price increase or a decrease!

**Formulae used in Israel for prices and wages**

Laspeyres’ index formula is used for prices. That is, we calculate monthly, the change in price of a fixed (or initial) basket of commodities.

The Unit index formula is used for wages. The unit in this case is the employee post. The cost per employee post is the total wages paid divided by the number of employee posts filled, and this is calculated monthly.
**Debt Linkage**

Regarding full linkage, the debt and debt repayments would rise and fall in proportion to some index, e.g. prices, wages, foreign currency, etc. This means that the new amount equals the original amount multiplied by the ratio of the new index value to the original index value.

If the linkage is partial, for example 99% linkage, then the values calculated using full linkage would be multiplied by 99/100.

**2 Accuracy and stability of index formulae**

An index formula should be both accurate and stable. "Stable" means that unrealistic prices should have little or no effect on the value of the formula. These matters are particularly important when debts are linked to an index formula.

**Accuracy**

We used a computer to simulate test situations where the true average index value is known. We observed that the Unit Index, the Normalized Unit index and Fisher’s index were accurate in all our tests, whereas the indices of Laspeyres and Paasche were accurate only when there was no correlation between price and quantity.

**Stability**

We noted in the full paper that the Unit index and Normalized unit index have a property which gives them good stability characteristics, namely, unrealistic prices are typically down weighted by low quantities. Laspeyres’, Paasche’s and Fisher’s indices do not have this property and therefore can have poor stability characteristics. The following two examples illustrate this.

**Example of a sudden price increase making a price unrealistic.**

Initial purchase: 2 kilos apples at 2 coins a kilo and 2 kilos bananas at 2 coins a kilo.

Most recent purchase: 4 kilos apples at 2 coins a kilo and 0 kilo bananas at 4 coin a kilo.

Here are the values of the index formulae.

<table>
<thead>
<tr>
<th>Index formula</th>
<th>Value</th>
<th>Laspeyres</th>
<th>Paasche</th>
<th>Fisher</th>
<th>Unit</th>
<th>Normalized Unit</th>
</tr>
</thead>
</table>
Example of a sudden price decrease because of an unrealistic price.

Initial purchase: 4 kilos apples at 2 coins a kilo and 0 kilos bananas at 4 coins a kilo.

Most recent purchase: 2 kilos apples at 2 coins a kilo and 2 kilo bananas at 2 coin a kilo.

Here are the values of the index formulae.

<table>
<thead>
<tr>
<th>Index formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laspeyres</td>
<td>100</td>
</tr>
<tr>
<td>Paasche</td>
<td>75</td>
</tr>
<tr>
<td>Fisher</td>
<td>87</td>
</tr>
<tr>
<td>Unit</td>
<td>100</td>
</tr>
<tr>
<td>Normalized Unit</td>
<td>100</td>
</tr>
</tbody>
</table>

Note that only the Unit index and Normalized Unit index were not affected by the unrealistic price in both these examples and had value 100 which indicates no change from the buyer's viewpoint.

**Section conclusion**

From the previous discussions we see that only the Unit index and the Normalized Unit index were both accurate and stable.

The Unit index can only be used to measure price change of a single item or similar items sold in the same units, and is in our opinion, the best choice for this case. The Normalized Unit index can be used to measure price change of different items sold in different units and is in our opinion the best choice for the general case. (See the full paper for further details.)

### 3 Wage and not price linkage

There are many formulae for calculating the price index of a number of different items. Fisher for example, gives one hundred and thirty four such formulae. He also compared these formulae using data for the U.S.A. covering the years 1913-1918. Except for the Unit index formula, he found that there was close agreement between the other four formulae presented here. His findings are different from ours. As the situations in the U.S.A. and Israel are significantly different from each other, we do not think that his findings are valid for Israel. One thing is certain. There is great controversy regarding the formula for calculating the price index of a number of different items. In view of this, debts should not be linked to such a price index. There is no justification that people should be harassed or have their property confiscated because of debts linked to such a price index, when there is so much controversy regarding its formula.

In the single item case (e.g. wages or price of manpower) there has not been controversy and the Unit index is the best choice.
accepted choice. In our opinion it is the correct formula in the single item case and we have seen that this index formula has good accuracy and stability characteristics. These and other reasons indicate that debts may be linked to an index of the average wage or income calculated by the Unit index formula.

4 Linkage without interest
For medium and large debts, the maximum long term linkage that the average wage earner can reasonably handle cannot be higher than the growth of his income. Any higher rate of linkage, will cause a larger and larger portion of the borrower's income to be used for debt repayments with the likelihood of inability to repay the debt. Similarly adding interest to this linkage will also increase the difficulty for the average wage earner to handle debt repayments and the inability of repaying the debt. Wide scale inability of borrowers to repay their debts, can cause the lender not to make a profit or in the worst case go bankrupt because of accumulating bad debts. We therefore see that this form of interest free linkage gives the maximum return which can be reasonably expected from personal loans and mortgages.

Furthermore, full linkage of debts to the average wage would cause hardship to about half the population - namely those whose rate of wage increase is below the average rate of wage increase. In view of this, it is proper to consider partial linkage to the average wage (e.g. 99%), in order that the majority of the population can handle the repayments.

Regarding the return to be expected from interest free loans linked to the published average gross wage, we have calculated from data for the years 1980 to 1986 as follows. One year fully linked loans would give a return of about 1.5% above the price index to the lender. (I have heard that this return is typical for Israel and not just for these years.) With 99% linkage, the lender can expect a return of about 0.5% above the price index. However, as prices and wages are measured by different formulae, the true return is likely to be higher (see full paper).

In view of the previous discussion, we suggest that personal loans and mortgages should at most be fully linked to the average wage or income and should bear no interest.

Note
The linkage rate on bank deposits can be set at a lower rate than on bank loans. For example, loans can be fully linked to the average wage, while deposits can be 99% linked to the average wage. As at present, a bank can profit from the difference of these two linkage rates.
5. Expressing the value of money in terms of time, encouraging honesty, discouraging fraud

Any sum of money can be expressed in terms of time, by dividing the sum of money by an average wage. For example, if your bank balance on a certain date is 1800 shekel and the average wage on that date is 1200 shekel a month, then the balance can be expressed as 1800/1200 = 1.5 average monthly wage, that is the value of 1.5 months of average labour.

We assert that balances and monies expressed in terms of an average wage gives a clear meaning of the value of these amounts in terms of what it means in time to an average man. This is a central idea of our approach for the linkage of debts and it can also be used for encouraging honesty and discouraging fraud. As we have written another article on this topic, we shall not discuss this matter further here. See http://homedir.jct.ac.il/~rafi/enc-dis.pdf

6. Linkage and economic stability

Debt linkage to wages is likely to behave as an automatic stabilizer (negative feedback), since when the standard of living improves, one effectively pays more, and when it decreases one effectively pays less. This stabilizes the spending power of borrowers. Timely repayment relief is given to the borrower at the expense of the lender when prices increase more than wages. The return to the lender is increased at the expense of the borrower when wages increase more than prices. So for example, a mortgage bank would receive a timely increase in its income when its wage bill increases.

(Linkage to prices can cause instability and this issue is discussed in the full paper.)

7. Others who have proposed linkage without interest or zero interest

John von Neumann developed a mathematical model to study economic equilibrium. This means that the proportional structure of the economy is not changing, but its size may change. He found that this can only occur when the interest rate equals the rate of expansion even if these rates are negative. In terms of linkage, this means that debts are linked to the size of the economy without interest. Perhaps John von Neumann is the first to consider interest free linkage even though he did not use this term.

Similarly in the economic model described by Milton Friedman, zero nominal interest rates are necessary for efficient resource allocation. Harold L. Cole and Narayana Kocherlakota further developed this approach.

However linkage without interest to the average wage is better suited for personal loans and mortgages, since it takes wages into account directly.

8. Other models of economic stability

The models of economic stability described in this document are not the only ones. Other models are also available, such as those developed by John von Neumann, Milton Friedman, Narayana Kocherlakota, and Harold L. Cole. These models provide alternative approaches to the study of economic stability and the management of debt in the economy.
Combined linkages?

It is possible to combine different linkages. For example, the debt can be linked without interest to the size of the economy and be repaid in instalments linked to the average wage. (With such an arrangement, the exact number of instalments may not be known at the time of giving the loan, though it would be known approximately.) Further investigation is needed on this topic.

8 Halachic questions

Firstly, a clarification. Regarding linkage, we earlier defined that the debt and debt repayments would rise and fall in proportion to some index, e.g. prices, wages, foreign currency, etc. Linkage in equal measure to both rises and falls is important as this avoids the prohibition of the biblical command of price linkage.

We now discuss a number of questions.

1) Does debt linkage to an inaccurate index formula contradict laws concerning just weights and measures, laws concerning price fraud, or the biblical requirement of a reciprocal work agreement?

In the book of Genesis we read regarding our forefather Abraham and Ephron the Hittite: "And Abraham hearkened to Ephron; and Abraham weighed to Ephron the silver, which he had named in the hearing of the children of Heth, four hundred shekels of silver, current money with the merchant". Thus we see that when making payment, the weighing scale was used as an instrument to measure the value of money, and regarding weights and measures the Torah demands accuracy similarly apply to the index formulae which are used for debt linkage. Surely accurate formulae must be used whenever debts are linked.

2) Are price linked loans or wage linked loans in keeping with the spirit of the biblical command of "...לזרב שammu אעיצמ אבב אבב"?

When prices increase more than wages the borrower pays more with price linkage and less with wage linkage. It therefore seems that wage linkage is in the spirit of the above biblical command, as less is taken from the borrower when he has less at his disposal. On the other hand, price linkage takes more from the borrower when he has less at his disposal and therefore seems to contradict the spirit of the above biblical command and can cause the borrower hardship.

3) Is linkage to the average wage akin to a reciprocal work agreement in which the works of both parties are of equal difficulty and therefore permissible?

Further investigation is needed on this topic.

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(3) Is linkage to the average wage akin to a reciprocal work agreement in which the works of both parties are of equal difficulty and therefore permissible?
Reciprocal work agreements, in which one party asks in return harder work from the other party, violate interest laws. If the works are of equal difficulty this is not prohibited. See 'order, Shir ha-Shakharim, 222' (and also 'order, Shir ha-Shakharim, 180'). So for example if the lender worked for one month in the field for the borrower in summer when it is dry and pleasant, and in return the borrower worked for one month in the field for the lender in winter when it is cold and raining, this would violate interest laws, as work in the field in cold rainy conditions is harder than work in the field in warm pleasant conditions. We also see from the table at the end of the article, that the price/wage ratio is generally lower in summer than in winter, which indicates that work in summer is generally easier than work in winter. In view of this, there is likely to be objection to lending one average monthly wage in summer and collecting one average wage in winter. However, there seems to be no objection to lending one average monthly wage in any month, and then collecting it in the same month in another year.

A better solution to this difficulty is for example, that the lender gives a loan of one average wage per month over one year and the borrower returns the loan paying one fifth of an average wage per month over five years. This is because one average wage of each of the months of the year is being lent and one average wage of each of the months of the year is being repaid. More generally, the loan can be provided in monthly instalments over one or more full years, and repaid in monthly instalments over one or more full years. This arrangement ensures that payments are made in terms of the average value of different kinds of labour of the whole country over one or more full years. Seasonal and regional fluctuations are thereby reduced and the risk is lowered.

Another better solution to this difficulty is to calculate an average wage from data for the previous twelve months. So if average wages over twelve month periods are used for linkage, then again one average wage of each of the months of the year can be lent in any month, and one average wage of each of the months of the year can be repaid in any month. Here there is no need to give or collect the loan in instalments. Here too it is ensured that payments are made in terms of the average value of different kinds of labour of the whole country over twelve month periods. Here too, seasonal and regional fluctuations are reduced and the risk is lowered.

(However, further investigation is needed to decide whether monthly instalments should be adjusted according to the number of days in a year or according to the number of work days in a year or perhaps no such adjustment is needed at all. For example suppose that adjustment is according to the number of work days and that there are 280 work days in the first year and 270, 290, 275, 285 and 280 work days in the five following years. Monthly instalments for the years following the first year would also be scaled by 280/270, 280/290, 280/275, 280/285 and 280/280 respectively.)

Another better solution is to calculate an average wage from data for the previous two years. So if average wages over two year periods are used for linkage, then again one average wage of each of the months of the year can be lent in any month, and one average wage of each of the months of the year can be repaid in any month. Here there is no need to give or collect the loan in instalments. Here too it is ensured that payments are made in terms of the average value of different kinds of labour of the whole country over two year periods. Here too, seasonal and regional fluctuations are reduced and the risk is lowered.

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280/280 respectively. Perhaps it is possible to avoid this problem by linking to an average hourly wage. See also question 8.)

4) Is linkage to the average wage akin to a partnership?

In a partnership, the partners share in profits and losses. Suppose that the loan is provided in monthly installments over one or more full years, and repaid in monthly installments over one or more full years. Suppose that linkage is both to rises and falls of the average wage (section 1). In that case borrower and lender share in the profits and losses caused by rises and falls of the average wage while the loan is being provided and repaid. So here there is similarity. The difference between this arrangement and a partnership is that the mutual contribution of the parties is not based on the value of their actual labours, but on the average cost of labour.

5) May wage linked loans be acceptable?

The following are very different views of two Chief Rabbis of Israel.

- The first view above accepts wage linkage is not to the detriment of the lender and in its income when its wage bill increases (section 1). In that case borrower and lender share in the profits and losses caused by rises and falls of the average wage. Linkage to the average wage or income and should bear no interest.

- The second view forbids the use of interest and permits the linkage of debts without interest. Therefore, it is possible to avoid this problem by linking to an average hourly wage.

6) Would the lender be getting a fair return on his money with wage linkage?

In section 3 of the full paper we presented evidence that wage linkage is not to the detriment of the lender and in section 4 we presented evidence that this form of interest free linkage gives the maximum return which can reasonably expected from personal loans and mortgages. When wages increase more than prices, the lender’s return is increased at the expense of the borrower. So for example, a mortgage bank would receive a timely increase in its income when its wage bill increases (section 6).

7) Are price linked loans near to profit and far from loss and what about wage linked loans?

- The first view above accepts wage linkage is both to rises and falls of the average wage. When wages increase more than prices, the lender’s return is increased at the expense of the borrower. So for example, a mortgage bank would receive a timely increase in its income when its wage bill increases (section 6).

- The second view forbids the use of interest and permits the linkage of debts without interest.
The data for the price index and the average monthly wage shows that during the period 1980 to 1986 the price index fell in value only once by about 1.3% whereas the average wage fell on eight occasions by about a few percent on each occasion. (See the table at the end of the article)

8) From an Halachic standpoint, which methods of calculating the average wage or income are acceptable for debt linkage? Which methods are to be preferred?

The unit index is used for measuring wages which seems a good choice. However, the published average monthly wage means the average gross wage per employee post of those employed. Should the net wage be used? Should those employed be included in this average with an income of zero or perhaps other income such as unemployment benefit be taken into account? Is it more correct to use the average hourly wage calculated using the formula: total gross salary paid nationally divided by total hours worked nationally? Is it more correct to use the average hourly wage calculated using the formula: total net salary received nationally divided by total hours worked nationally? Should average monthly income be used? Should average annual income be used? Should average taxable income be used? Etc.

It seems to us that the unemployed should be included in the calculation. It also seems to us that linkage to average net income is best in agreement with the commandment of "...and to the right no...", and income from all sources should be taken into account.

9) From an Halachic standpoint, which formulae for calculating price indices are acceptable for debt linkage? Which formulae are to be preferred?

There are many formulae published in the literature for the measurement of price change of which the formulae of Laspeyres, Paasche, Fisher and the Normalized Unit Index are a few. The indices of Laspeyres and Paasche performed poorly on our computational assessment, but the Normalized Unit Index and the index of Fisher performed consistently well. However there are other considerations for preferring the Normalized Unit index formula to Fisher's index formula (section 2). In addition, there is great controversy regarding the formula for calculating the price index of a number of different items (section 3). We therefore suggest that debts should not be linked to such a price index.

10) Does the prohibition of "א א" have Halachic implications regarding the choice of index formula for debt linkage?

(Without knowledge as to how index formulae are calculated, the discussion here will not be understood. Such a reader should skip this question until he has read the table at the end of the article.)
the appendix.) It seems to us that linkage to the formula of Laspeyres or of Paasche violate this prohibition as both these formulae compare the price of a “basket” of commodities which may or may not be available in the market. It also seems to us that debt linkage to Laspeyres’ index would be favourable to the lender whereas linkage to Paasche’s index is favourable to the borrower. As the index of Fisher is the geometric mean of the index of Laspeyres and Paasche, use of this index for debt linkage may also violate this prohibition. The Unit index formula does not seem to violate this prohibition as the cost per unit is being compared based on what is actually available in the market. Similarly, the Normalized unit index does not seem to violate this prohibition and this depends on how "normalized unit" was defined.

11) Which forms of debt linkage are Halachically acceptable when lending to the non-Jew?

Though the Torah permits taking interest from a non-Jew, there was a period in Jewish history where the sages limited who may take interest from a non-Jew or under what conditions this interest may be taken, see [51x64]_ Halachic difficulty discussed in question (3) above. In our opinion, wage linkage considerations are also relevant when lending with linkage. These considerations are also relevant when lending with linkage to non-Jews and in our opinion the debt linkage policy for such lending, should be morally justifiable and defensible by clear reasoned argument. In our opinion, wage linkage meets these criteria and therefore is to be preferred for such lending. Furthermore, the first question in this section is equally relevant whether lending to Jew or non-Jew and needs to be taken into account here.

12) See the Hebrew text for a discussion of this matter.

**The use of a twelve month period**

This is an important technique used for dealing with the Halachic difficulty discussed in question (3) above. In addition this technique reduces the risk, avoids problems of seasonality, and contributes to stability of an average wage, (and similarly for a price index, a foreign currency rate etc.).

9 Conclusions and suggestions

We have seen there are several reasons for using linkage to wages and not to prices for loans and other transactions.

We have also seen that interest free wage linkage can give a profit to the lender and is not oppressive to the borrower and so should be used for personal loans and mortgages. We have also explained how the loan should be given so as to meet Halachic requirements and reduce the risk.
Furthermore, the linkage rate on bank deposits can be at a lower linkage rate than on bank loans. As at present, a bank can profit from the difference of these two linkage rates.

Detailed lists of conclusions and suggestions appear in the full paper.

Appendix - A layman's guide to index formulae

We shall explain by means of a simple example how price and wage indices are calculated. Let us say a family wishes to measure the change in the price of fruit and records details of the "fruit basket" it purchases as follows.

Initial purchase:
3 kilos apples at 3 coins a kilo and 5 kilos bananas at 2 coins a kilo.

Most recent purchase:
4 kilos apples at 2 coins a kilo and 1 kilo bananas at 4 coins a kilo.

There are several ways of measuring the change in the family's price index of fruit from the initial to the most recent purchase. We give five methods corresponding to five kinds of indices.

Method 1 - (Laspeyres' Index)

The fruit basket at the initial purchase consisted of 3 kilos apples and 5 kilos bananas. At the time of the initial purchase this basket costs 3×3+5×2=19 coins. At the time of the most recent purchase this basket would have cost 3×2+5×4=26 coins. The change expressed as a percentage ratio is 26/19×100=136.8. This is the value of Laspeyres' index at the most recent purchase.

Method 2 - (Paasche's Index)

The fruit basket at the most recent purchase consisted of 4 kilos apples and 1 kilo bananas. At the time of the initial purchase this basket would have cost 4×2+1×4=12 coins. At the time of the most recent purchase this basket cost 4×2+1×4=12 coins. The change expressed as a percentage ratio is 12/14×100=85.7. This is the value of Paasche's index at the most recent purchase.

Method 3 - (Fisher's Index)

Fisher's index is simply the geometric mean (square root of the product) of Laspeyres' and Paasche's indices, that is \( \sqrt{(136.8 \times 85.7)} = \sqrt{11723.76} = 108.3 \).
Method 4 - (Unit Index)

From the family's viewpoint the cost per kilo of fruit at each purchase is the total amount paid divided by the total kilos bought.

At the initial purchase this is \((3\times3+5\times2)/(3+5)=19/8=2.375\) coins per kilo.

At the most recent purchase this is \((4\times2+1\times4)/(4+1)=12/5=2.400\) coins per kilo.

The change expressed as a percentage ratio is \(2.400/2.375\times100=101.1\). This is the value of the Unit index at the most recent purchase. (What we have done is calculate the cost per "unit" of fruit, the unit in this case being the kilo). The Unit index is a natural way of measuring change if we restrict ourselves to a single item or similar items which are sold in the same units (see also [11]). However, in the above form it can not be used for measuring price change of different items sold in different units.

Method 5 - (Normalized Unit Index)

To use the unit index in a general situation where we wish to determine the change in price of different items such as food, clothing, furniture, etc. we have to in a certain sense equate all these items in a natural way. One way of doing this is to say that one "normalized unit" of an item, is the amount of that item that can be purchased for one unit of currency based on its mean price for the purchases compared. We now repeat the previous calculation but will treat apples and bananas as different items

For apples this is \((3\times3+4\times2)/(3+4)=17/7\) coins per kilo.

For bananas is \((5\times2+1\times4)/(5+1)=14/6\) coins per kilo.

Therefore the quantity of apples which can be purchased for one coin is \(7/17\) kilo and this is the normalized unit for apples.

Similarly the quantity of bananas which can be purchased for one coin is \(6/14\) kilo and this is the normalized unit for bananas.

We now calculate for each purchase the total amount paid divided by the total units bought. This gives the cost per normalized unit.

At the initial purchase we have:

Total cost for 3 kilos apples and 5 kilos bananas is 19 coins (as before).
Total units bought is
\[ \frac{3}{7/17} + \frac{5}{6/14} = 3 \times \frac{17}{7} + 5 \times \frac{14}{6} = 18.952. \]
The cost per unit is \( 19/18.952 = 1.003 \) coins per normalized unit.

Similarly, at the most recent purchase we have:
Total cost for 4 kilos apples and 1 kilo bananas is 12 coins (as before).
Total units bought is
\[ \frac{4}{7/17} + \frac{1}{6/14} = 4 \times \frac{17}{7} + 1 \times \frac{14}{6} = 12.048. \]
The cost per unit is \( 12/12.048 = 0.996 \) coins per normalized unit.

The change in the cost per unit when expressed as a percentage ratio is \( 0.996/1.003 \times 100 = 99.3 \). This is the value of the Normalized Unit index at the most recent purchase.

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My internet page: http://homedir.jct.ac.il/~rafi
Link to the full paper
http://homedir.jct.ac.il/~rafi/ifwl.pdf
Revised October 2018 - תורן ינשע"ט.
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