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# COMPUTATIONAL TESTS AS A MEANS OF ASSESSING THE INACCURACIES OF INDEX NUMBERS

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ABSTRACT: In this paper we propose using computational tests for appraising the inaccuracies of index numbers (for example, price and wage indices). This is done by programming a hypothetical situation where the true average index value is known and the variation of prices and quantities and the relation between them can be chosen. The indices are then calculated using the formulae of Laspeyres, Paasche, Fisher, the Unit index and Normalized Unit index formulae. By comparing the average values of the above formulae with the true average index value, we are able to obtain a quantitative indication of the errors in these formulae under various situations. The Normalized Unit and Fisher index formulae were found to be consistently accurate for all our tests whereas the formulae of Laspeyres and Paasche were found to have very poor accuracy in the majority of our tests. The Unit index was also consistently accurate in all our tests but its use is appropriate in the single item case thus making this formula suitable for measuring wages. In our informal discussion of stability we indicated that the Normalized unit index and the Unit index have good stability characteristics whereas the indices Laspeyres, Paasche and Fisher have poor stability characteristics. In view of the above the Normalized Unit index is the best of the formulae we have discussed for measuring prices (many item case), and the Unit index is the best of the formulae we have discussed for measuring wages (single item case).

KEYWORDS: index number, error, computational test.

NOTES: An earlier version of this paper has been published in the IMA Journal of Mathematics Applied in Business and Industry, Vol. 3 No. 2 pp. 141-152, 1991. There is a spelling error in the published paper, where we repeatedly wrote "Fischer" instead of "Fisher". This has been corrected here. Section 2.3 is a recent addition.

CONTENTS

## 1. INTRODUCTION

### 1.1 A Layman's Guide to Index Formulae

To enable the lay reader to understand this paper, we first explain by means of a simple example how price and wage indices are calculated. Let us say a family wishes to measure the change in the price of fruit and records details of the "fruit basket" it purchases as follows.

Fruit basket - initial purchase
- 3 kilos apples at 3 coins a kilo
- 5 kilos bananas at 2 coins a kilo

Fruit basket - most recent purchase
- 4 kilos apples at 2 coins a kilo
- 1 kilo bananas at 4 coin a kilo

There are several ways of measuring the change in the family's price index of fruit from the initial to the most recent purchase. We give five methods corresponding to five kinds of indices.

Method 1 - (Laspeyres' Index)

The fruit basket at the initial purchase consisted of 3 kilos apples and 5 kilos bananas. At the time of the initial purchase this basket costs $3\times3 + 5\times2 = 19$ coins. At the time of the most recent purchase this basket would have cost $3\times2 + 5\times4 = 26$ coins. The change expressed as a percentage ratio is $26/19 \times 100 = 136.8$ . This is the value of Laspeyres' index at the most recent purchase.

Method 2 - (Paasche's Index)

The fruit basket at the most recent purchase consisted of 4 kilos apples and 1 kilo bananas. At the time of the initial purchase this basket would have cost $4\times3 + 1\times2 = 14$ coins. At the time of the most recent purchase this basket cost $4\times2 + 1\times4 = 12$ coins. The change expressed as a percentage ratio is $12/14 \times 100 = 85.7$ . This is the value of Paasche's index at the most recent purchase.

Method 3 - (Fisher's Index)

Fisher proposed comparing the geometric mean (square root of the product) of the costs of the initial and most recent basket at initial prices with the geometric mean of the costs of the initial and most recent basket at most recent prices. That is the initial geometric mean $\sqrt{(14\times19)} = 16.30$ is compared with the most recent geometric mean $\sqrt{(26\times12)} = 17.66$. The change expressed as a percentage ratio is $(17.66/16.30)\times100 = 108.3$ which is the value of Fisher's index at the most recent purchase. Fisher's index also equals to the geometric mean (square root of the product) of Laspeyres' and Paasche's indices, namely $\sqrt{(136.8 \times 85.7)} = \sqrt{11723.76} = 108.3$.

## Method 4 - (Unit Index)

From the family's viewpoint the cost per kilo of fruit at each purchase is the total amount paid divided by the total kilos bought.

At the initial purchase this is :-

$(3\times3 + 5\times2) / (3 + 5) = 19/8 = 2.375$ coins per kilo.

At the most recent purchase this is :-

$(4\times2 + 1\times4) / (4 + 1) = 12/5 = 2.400$ coins per kilo.

The change expressed as a percentage ratio is $2.400/2.375 \times 100 = 101.1$ . This is the value of the Unit index at the most recent purchase. (What we have done is calculate the cost per "unit" of fruit, the unit in this case being the kilo).

## Method 5 - (Normalized Unit Index)

The Unit index is a natural way of measuring change if we restict ourselves to one item or similar items which are sold in the same units. To use it in a more general situation where we wish to determine the change in price of different items such as food, clothing furniture etc. we have to in a certain sense equate all items in a natural way. One way of doing this is to say that one "normalized unit" of an item, is the amount of that item that can be purchased for one unit of currency based on its mean price for the purchases compared. We can now calculate as follows and will treat apples and bananas as if they were separate commodities.

The mean price of apples over the two purchases is the total amount paid for the apples purchased divided by the total amount of apples purchased that is:

$(3\times3 + 4\times2) / (3 + 4) = 17/7$ coins per kilo

Similarly the mean price for bananas is:

$(5\times2 + 1\times4) / (5 + 1) = 14/6$ coins per kilo

Therefore the quantity of apples which can be purchased for one coin is 7/17 kilo and this is the normalized unit for apples.

Similarly the quantity of bananas which can be purchased for one coin is 6/14 kilo and this is the normalized unit for apples.

We now calculate for each purchase the total amount paid divided by total units bought which gives the cost per normalized unit.

At the initial purchase we have :-

Total cost for 3 kilos apples and 5 kios bananas is of course as before, namely 19 coins.

Total units bought is $3/(7/17) + 5/(6/14) = 3\times(17/7) + 5\times(14/6) = 18.952$

The cost per unit $19/ 18.952 = 1.003$ coins per normalized unit.

At the most recent purchase we similarly have:-
Total cost for 4 kilos apples and 1 kilo bananas is as before, namely 12 coins.
Total units bought is 4/(7/17) + 1/(6/14) = 4×(17/7) + 1×(14/6) = 12.048
The cost per unit is 12/ 12.048 = 0.996 coins per normalized unit.

The change in the cost per unit when expressed as a percentage ratio is 0.996 /1.003 × 100 = 99.3 . This is the value of the Normalized Unit index at the most recent purchase.

Some General Comments:

1) Mathematical formulae for various indices are given in appendix A.

2) Method 1 (Laspeyres' index) is used for prices. That is, we calculate monthly, the change in price of a fixed (or initial) basket of commodities.

3) Method 4 (the Unit index) is used for wages. The unit in this case is the employee post. The cost per employee post is the total wages paid divided by the number of employee posts filled, and this is calculated monthly.

4) Methods 2,3,5 (Paasche's Fisher's and the Normalized Unit index) are not used generally. In this paper they are used in conjunction with the other formulae, to understand the errors and uncertainties in indexation.

5) As can be seen from the above example different methods of calculation can give different index values, the greatest difference being between the indices of Laspeyres and Paasche.

6) We would mention that "chaining" [1,2,3] can be used to improve the characteristics of various indices and this is briefly discussed later on in the paper.

## 1.2 A Pragmatic Assessment of the Accuracy of Index Numbers

Table 1 presents a comparison of Laspeyres and Paasche Price indices (with and without chaining). These data are for the United Kingdom and covers the years 1958 to 1967 when inflation was running between 1% to 5% a year (or about 25% for ten years). The results show that for a large economy running at low inflation there is reasonable agreement between the formulae of Laspeyres and Paasche with about a 5% difference between these formulae after 25% inflation in ten years. However whether agreement between these formulae can be expected for a small economy running at high inflation is questionable to say the least. Bear in mind that some countries experienced inflation of 25% within one or two months not in ten years. Can a 5% difference between these formulae be expected every one or two months in these countries and what differences can be expected with an accumulated inflation rate of hundreds or a thousand percent ?

## TABLE 1 - COMPARISON OF LASPEYRES & PAASCHE PRICE INDICES. (U.K.)

| Year | Expenditure per Household | Laspeyres | Paasche | Chain Laspeyres | Chain Paasche |
|---|---|---|---|---|---|
| 1958 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| 1959 | 104.31987 | 100.85823 | 100.54480 | 100.85823 | 100.54480 |
| 1960 | 109.14066 | 101.74241 | 101.11714 | 101.72658 | 101.12728 |
| 1961 | 115.07212 | 105.03774 | 103.76592 | 104.63828 | 103.85400 |
| 1962 | 117.48085 | 109.33042 | 107.86972 | 108.53627 | 107.63539 |
| 1963 | 126.63335 | 111.71473 | 108.43431 | 110.67846 | 109.39957 |
| 1964 | 127.79486 | 115.58460 | 112.41367 | 114.23699 | 112.62434 |
| 1965 | 143.59308 | 121.22315 | 116.13442 | 120.25972 | 117.84037 |
| 1966 | 147.08847 | 126.08628 | 120.82135 | 124.86262 | 122.30600 |
| 1967 | 153.92306 | 129.44595 | 123.40484 | 127.84142 | 125.42371 |

The above table is taken from [4].

### 1.3 Mathematical Tests for Qualitatively Assessing Index Numbers

Many formulae have been published in the literature for measuring inflation and we have illustrated five of them. The previous discussion shows there is uncertainity in determining the index value but it does not indicate which formula is to be preferred. How then do we compare the quality of the formulae to determine which are better ? Mathematical tests have been proposed [1,2,3] and we mention some of them here. Let $I_{bi}$ be any formula for the index in month i with respect to month b (a base month). We assume $I_{bi}$ is expressed as a pure ratio not as a percentage ratio (i.e. regarding these tests, the index value in the base month is 1 and not 100 as is usual). With this in mind then regardless of the formula used the following properties should hold for months b,i,k.

1) $I_{ii} = 1$.
2) $I_{bi} = 1/ I_{ib}$ for $b \neq i$.
3) $I_{bi} = I_{bk} \times I_{ki}$ for $b < k < i$
4) The value of $I_{bi}$ should be independent of the units in which quantities are expressed.

(The restrictions imposed on properties 2,3 above, make the first three conditions independent of each other. Properties 1,2 can in fact be derived from an unrestricted version of property 3. Similarly property 1 can be derived from an unrestricted version of property 2.)

Regarding these tests, the formulae of Laspeyres and Paasche satisfy properties 1 and 4, while the formula of Fisher satisfies properties 1,2, and 4. The unit index satisfies properties 1,2,3,4 when used to measure the change in price of one item sold from many shops but it only satisfies properties 1,2,3 if used to measure the change in price of many items. The normalized unit index satisfies properties 1,2,4. (It satisfies property 4 because normalized unit is defined in terms of average price of an item and it does not depend on the actual units used to sell an item.)

Actually, from any index formula $I_{bi}$, we can derive a formula which satisfies conditions 1,2,3. To derive a formula to satify conditions 1,2 instead of $I_{bi}$, we use $G_{bi} = \sqrt{(I_{bi}/ I_{ib})}$. Intuitively, what we have done is take to take the geometric mean of two estimates of the index from month b to i, namely $I_{bi}$ and $1/I_{ib}$. Incidentally, if we apply this transformation to the index of Laspeyres or to the index of Paasche, we derive in fact Fisher's index as can be easily verified.

Regarding deriving a formula satisying property 3, chaining [1,2,3] may be used. Namely, instead of $I_{bi}$ use $C_{bi} = I_{b,b+1} x\ I_{b+1,b+2} x....x\ I_{i-1,i}$.

In short by using both these techniques we can always derive a formula satisfying properties 1,2,3.

## 2. QUANTATIVE ASSESSMENT OF THE INACCURACIES OF INDEX FORMULAE AND AN INFORMAL DISCUSSION OF THEIR STABILITY

There is a major problem of giving a quantitative indication of the accuracy of index formulae in that one would need to know in advance the true index value or inflation rate, which is impossible with real world data. However by using a computer to simulate a situation where the true average index value is known, we can get a clearer picture of the inaccuracies of these formulae. In this section we present this approach and return to using the accepted practice that the index value in the base month is 100.

### 2.1 The Use of Computational Tests

In Table 2 we present a comparison of Laspeyres, Paasche, Fisher, Unit and Normalized Unit Indices. These results are produced by a computer program. We simulate a hypothetical situation where there are a number of shops selling the same item. The situation in any month is independent of the situation in other months. Except for random variation, prices and quantities are not changing. Prices, which are the independent variables, are programmed to vary randomly about a fixed midpoint with known random variation. The table gives data for five different kinds of relationship between quantity and price namely, (1) quantity inversely proportional to price squared, (2) quantity inversely proportional to price, (3) no correlation between quantity and price, (4) quantity directly proportional to price, (5) quantity directly proportional to price squared. Regarding the quantity variation shown in table 2, we would stress that this is the random variation in the quantity level which is independent of and not related to the price variation. Similarly, the price variation shown in table 2, is the random variation in the price level which is independent of and not related to the quantity variation. (Mathematical formulae, details of the simulation, as well as the program and its output are given in the appendices.)

Apart from random variation, prices and quantities are not changing. Hence on the average, the index should remain 100 which is therefore the true average index value. We use the formulae of Laspeyres, Paasche, Fisher, the

Unit Index and the Normalized Unit index to calculate these indices and compare these calculated values with the pre-programmed value of 100*. The **mean deviation per measurement**# from this pre-programmed value is calculated and given in the table. (For a more general treatment and a mathematical explanation see [5], in particular the section "The symmetric case - The true geometric expected index value is known".)

Now, regarding the data from Table 2, we firstly observe that in all the tests, the deviations from the expected value are not significantly changed by the independent quantity variation. We see that when there is no correlation between quantity and price, all five formulae agree with the pre-programmed index value to within about 0.1% per measurement regardless of the price variation. On the other hand when the quantity is inversely proportional to price (or to price squared), we see very poor accuracy in the formulae of Laspeyres (too high) and Paasche (too low) as the price variation increases. However the Unit index, and the Normalized Unit index agree with the pre-programmed index value to within about 0.1% per measurement regardless of the price variation and Fisher's index to within about 0.2%. Similarly, in the case when quantity is directly proportional to price (or to price squared), we see very poor accuracy in the formulae of Laspeyres (too low) and Paasche (too high) as the price variation increases. However Fisher's index, the Unit index, and the Normalized Unit index agree with the pre-programmed index value to within about 0.05% per measurement regardless of the price variation.

In addition as it is unusual that the elasticity of demand is positive, it is probable that in practice the index of Laspeyres will be higher than the true index value and the index of Paasche to be lower than the true index value. This deduction is consistent with the data for the U.K. presented in Table1 where Laspeyres' index is consistently higher than the index of Paasche.

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* There is no loss in generality of having a fixed programmed index value, as the errors of index measurement are caused by variations in the price and quantity levels of a commodity and these variations can be altered by the user of the program. (Just multiplying all price and quantity levels by a constant factor causes no error in the value of any of the formulae presented.)

# The mean deviation per measurement is calculated from the geometric mean of the error factors (error ratios) of the calculated index with respect to the true average index value. This geometric mean of the error factors is then converted to a percentage error. The reason we make the calculation in this way and not using the arithmetic mean is that these errors typically accumulate multiplicatively and not additively. For example if we have an error of +50% in measuring the change from month 1 to month 2 followed by an error of -50% from month 2 to month 3, the combined error is not 50%-50% = 0% but -25%. This is clearly seen from the error factors which are 1.5 and 0.5. So the combined error factor is 1.5×0.5=0.75 which as a percentage is 100×(0.75-1) = -25%. The mean error per month should be calculated from the geometric mean of the error factors namely $\sqrt{(1.5\times0.5)} = \sqrt{0.75} = 0.866$ which as a percentage is -13%. We would further mention that in a previous version of this paper we used the arithmetic mean of the error deviations which in fact gave similar results to the results obtained using the geometric mean, probably due to the fact that the that the number of shops is large.

In short, we see that, the Unit Index, the Normalized Unit index and the index of Fisher give consistently accurate measurements of the true index value under all our tests whereas the indices of Laspeyre and Paasche gave accurate results only in the case where there is no correlation between price and quantity.

## 2.2 Some Clarifying Remarks

Some words of clarification are in order regarding the direction we have taken and also what we have and what we have not simulated.

1) The prime aim of this paper is to give a quantitative indication of the errors of various index formulae and this is done by simulating a variety of situations where the true average index is known. The simulations do not constitute an economic model but we believe we have included sufficient detail in the simulations to appraise the inaccuracies of index formulae (after all we are able to specify the relationship between price and quantity levels and the independent variation in price and quantity levels). We will draw an analogy from the physical sciences in support of using such a simplification. In order test a scale for weighing human beings, one uses standard test weights; no one would suggest using "standard human beings". Obviously, the more test weights used the greater the confidence in the scale. Similarly to test indices for accuracy, test cases where the true average index is known seems to us a valid approach for gaining a quantitative indication of the potential errors of index formulae; the use of a realistic economic model not being an essential requirement.

2) The approach we have presented has a practical computational bias and the economic side is not discussed. In order to develop the economic side, it would be necessary to develop a model whose true index is known which is more realistic economically. Further work is needed to develop this aspect.

3) In the simulations, we only treated the single item case in which prices are the independent variable. In [5], the many item and single item cases are discussed. There, prices may be the independent variables or quantities may be the independent variables or prices and quantities may depend on a third parameter. The results we got there are similar to those given here. The approach taken there is mathematical, using probability theory.

## 2.3 An informal discussion of the stability of index formulae

Both the Unit index and Normalized unit index have the property of always multiplying price and quantity of an item in the same month. (Note the form of the price and quantity products in their formulae in Appendix A.) This property will give both these indices good stability characteristics since unrealistic prices will typically be down weighted by low quantities. Laspeyres', Paasche's and Fisher's indices do not have this property and therefore may have poor stability characteristics.

Also note that Fisher's index is defined as the geometric mean of two inaccurate indices, and this may give it poor stability characteristics. Perhaps in other situations to those we have discussed here, the errors of these two indices will not cancel out and so Fisher's index may be unreliable.

The following two examples, illustrate what we mean.

*Example of a sudden price increase making a price unrealistic.*

Initial purchase: 2 kilos apples at 2 coins a kilo and 2 kilos bananas at 2 coins a kilo.
Most recent purchase: 4 kilos apples at 2 coins a kilo and 0 kilo bananas at 4 coin a kilo.

Here are the values of the index formulae.

| Index formula | Value |
|---|---|
| Laspeyres | 150 |
| Paasche | 100 |
| Fisher | 122 |
| Unit | 100 |
| Normalized Unit | 100 |

*Example of a sudden price decrease because of an unrealistic price.*

Initial purchase: 4 kilos apples at 2 coins a kilo and 0 kilos bananas at 4 coins a kilo.
Most recent purchase: 2 kilos apples at 2 coins a kilo and 2 kilo bananas at 2 coin a kilo.

Here are the values of the index formulae.

| Index formula | Value |
|---|---|
| Laspeyres | 100 |
| Paasche | 75 |
| Fisher | 87 |
| Unit | 100 |
| Normalized Unit | 100 |

Note that only the Unit index and Normalized Unit index were not affected by the unrealistic price in both these examples and had value 100 which indicates no change from the buyer's viewpoint.

# TABLE 2 - COMPARISON OF LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES FROM COMPUTER GENERATED DATA

Quantity inversely proportional to price squared (ELASTICITY = -2)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | 2.7615 | -2.6800 | 0.0038 | 0.0044 | 0.0038 |
| 40.0 | 0.0 | 12.3871 | -11.0060 | 0.0088 | 0.0119 | 0.0081 |
| 60.0 | 0.0 | 35.1909 | -26.0042 | 0.0177 | 0.0254 | 0.0112 |
| 80.0 | 0.0 | 101.9879 | -50.4549 | 0.0375 | 0.0573 | 0.0109 |

Quantity inversely proportional to price (ELASTICITY = -1)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | 1.3675 | -1.3420 | 0.0035 | 0.0039 | 0.0036 |
| 40.0 | 0.0 | 5.9095 | -5.5651 | 0.0078 | 0.0094 | 0.0078 |
| 60.0 | 0.0 | 15.5090 | -13.4028 | 0.0138 | 0.0181 | 0.0128 |
| 80.0 | 0.0 | 37.2726 | -27.1147 | 0.0258 | 0.0361 | 0.0183 |

No correlation between quantity and price (ELASTICITY = 0)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 |
| 40.0 | 0.0 | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0065 |
| 60.0 | 0.0 | 0.0098 | 0.0098 | 0.0098 | 0.0098 | 0.0098 |
| 80.0 | 0.0 | 0.0130 | 0.0130 | 0.0130 | 0.0130 | 0.0130 |

Quantity directly proportional to price (ELASTICITY = 1)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | -1.3107 | 1.3341 | 0.0030 | 0.0027 | 0.0030 |
| 40.0 | 0.0 | -5.0507 | 5.3303 | 0.0052 | 0.0038 | 0.0051 |
| 60.0 | 0.0 | -10.6944 | 11.9898 | 0.0065 | 0.0033 | 0.0059 |
| 80.0 | 0.0 | -17.5580 | 21.3147 | 0.0071 | 0.0012 | 0.0050 |

Quantity directly proportional to price squared (ELASTICITY = 2)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | -2.5574 | 2.6299 | 0.0026 | 0.0021 | 0.0027 |
| 40.0 | 0.0 | -9.1794 | 10.1154 | 0.0037 | 0.0016 | 0.0041 |
| 60.0 | 0.0 | -17.6250 | 21.4049 | 0.0036 | -0.0006 | 0.0045 |
| 80.0 | 0.0 | -25.9931 | 35.1309 | 0.0031 | -0.0029 | 0.0052 |

# TABLE 2 CONTINUED - COMPARISON OF LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES FROM COMPUTER GENERATED DATA

Quantity inversely proportional to price squared (ELASTICITY = -2)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 80.0 | 2.7735 | -2.6534 | 0.0233 | 0.0055 | 0.0191 |
| 40.0 | 80.0 | 12.4260 | -10.9466 | 0.0596 | 0.0139 | 0.0397 |
| 60.0 | 80.0 | 35.2873 | -25.9080 | 0.1185 | 0.0273 | 0.0606 |
| 80.0 | 80.0 | 102.1881 | -50.3198 | 0.2235 | 0.0534 | 0.0724 |

Quantity inversely proportional to price (ELASTICITY = -1)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 80.0 | 1.3770 | -1.3176 | 0.0206 | 0.0050 | 0.0190 |
| 40.0 | 80.0 | 5.9366 | -5.5142 | 0.0475 | 0.0116 | 0.0394 |
| 60.0 | 80.0 | 15.5702 | -13.3260 | 0.0846 | 0.0214 | 0.0627 |
| 80.0 | 80.0 | 37.4116 | -27.0142 | 0.1454 | 0.0394 | 0.0920 |

No correlation between quantity and price (ELASTICITY = 0)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 80.0 | 0.0101 | 0.0258 | 0.0180 | 0.0044 | 0.0188 |
| 40.0 | 80.0 | 0.0205 | 0.0514 | 0.0359 | 0.0088 | 0.0375 |
| 60.0 | 80.0 | 0.0310 | 0.0766 | 0.0538 | 0.0131 | 0.0561 |
| 80.0 | 80.0 | 0.0416 | 0.1016 | 0.0716 | 0.0174 | 0.0747 |

Quantity directly proportional to price (ELASTICITY = 1)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 80.0 | -1.3065 | 1.3550 | 0.0154 | 0.0038 | 0.0181 |
| 40.0 | 80.0 | -5.0487 | 5.3705 | 0.0254 | 0.0059 | 0.0326 |
| 60.0 | 80.0 | -10.7012 | 12.0501 | 0.0297 | 0.0060 | 0.0393 |
| 80.0 | 80.0 | -17.5788 | 21.3968 | 0.0283 | 0.0043 | 0.0338 |

Quantity directly proportional to price squared (ELASTICITY = 2)

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 80.0 | -2.5559 | 2.6493 | 0.0129 | 0.0031 | 0.0171 |
| 40.0 | 80.0 | -9.1866 | 10.1509 | 0.0159 | 0.0034 | 0.0256 |
| 60.0 | 80.0 | -17.6457 | 21.4540 | 0.0113 | 0.0016 | 0.0226 |
| 80.0 | 80.0 | -26.0260 | 35.1883 | 0.0021 | -0.0007 | 0.0161 |

The results in table 2 are abstracted from the runs giving summary output in appendix D which appertains to 1000 shops being simulated for 1000 cycles. The cumulative results for 1000 cycles are presented here. Only results where the independent quantity variation is zero or eighty are shown. Appendix D contains similar results for other values of the independent quantity variation. As can be seen from this table or from Appendix D, there is only a slight change in the values of the mean deviations of the various indices for different values of the independent quantity variation. In other words, these deviations depend primarily on the independent price variation, the independent variable of the simulation

## 3 CONCLUSION

In view of the results presented, we conclude that the values of the Laspeyres and Paasche indices are likely to give serious errors in times of high price instability. In the informal discussion of stability, we presented examples in which the indices Laspeyres, Paasche and Fisher have poor stability characteristics. However, the Normalized Unit index is expected to give reliable index values in times of high price instability, in view of their consistently accurate measurements under all our tests and their good stability characteristics. The Unit index was also consistently accurate in all our tests and has good stability characteristics. However, its use is appropriate only in the single item case which makes this formula suitable for measuring wages.

It is a pity that in practice, price indices are measured using the formulae of Laspeyres (and Paasche), formulae which we have shown to have a potential for serious error and likely to have poor stability characteristics. Great strides have been made in the fields of computation and data collection since the formulae of Laspeyres and Paasche were proposed, making it much easier to use the more accurate and stable formulae in practice. The Normalized Unit index was accurate and stable and is the best of the formulae we have discussed for measuring prices (many item case). The Unit index was accurate and stable and is the best of the formulae we have discussed for measuring wages (single item case).

## 4 ACKNOWLEDGMENT

The author would like to thank the editor and the referees for their constructive comments and criticisms and the editorial staff for their careful help.

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# INTRODUCTION TO THE APPENDICES

In order that the paper should be understandable to as wide a readership as possible we have minimized mathematical details or a description of the computer programs and algorithms in the body of the paper. In the appendices, we give these details as well as discuss some auxiliary issues.

The appendices give the following information.

A) Various index formulae (including the Normalized Unit Index).
B) Mathematical and computational details of the simulation.
C) Listing of the computer program.
D) Output from the computer program. Seven runs are given. Two runs gives full output on a month by month basis as well as summary tables which give results cycle by cycle cumulatively. The full output feature was used for testing and debugging the program and we include this output to facilitate others to check our work. Table 2 is summarized from these long runs which give the average deviations of the various indices from their programmed value 100.0 after the final cycle. The deviation of the mean value from the expected value 1.0 of the random numbers used is also reported so that the quality of the random numbers can be assessed.

## APPENDIX A - Mathematical Formulae

i - month number (base month is month b)
j - item number
$p_{ij}$ - price in month i of item j
$q_{ij}$ - quanity sold in month i of item j
$L_{bi}$ - Laspeyres' price index in month i with respect to base month b
$P_{bi}$ - Paasche's price index in month i with respect to base month b
$F_{bi}$ - Fisher's price index in month i with respect to base month b
$U_{bi}$ - Unit (Wages) index in month i with respect to base month b
$N_{bi}$ - The Normalized Unit index in month i with respect to base month b

$$L_{bi} = \frac{\Sigma_j \; p_{ij} \, q_{bj}}{\Sigma_j \; p_{bj} \, q_{bj}} \times 100$$

$$P_{bi} = \frac{\Sigma_j \; p_{ij} \, q_{ij}}{\Sigma_j \; p_{bj} \, q_{ij}} \times 100$$

$$F_{bi} = \frac{\sqrt{( \Sigma_j \; p_{ij} \, q_{ij} \; \times \; \Sigma_j \; p_{ij} \, q_{bj} )}}{\sqrt{( \Sigma_j \; p_{bj} \, q_{ij} \; \times \; \Sigma_j \; p_{bj} \, q_{bj} )}} \times 100 = \sqrt{(L_i \times P_i)}$$

The previous formulae for $L_{bi}$, $P_{bi}$ and $F_{bi}$ can be used to measure price change of a set of different items. The next formula $U_{bi}$ the unit index is only useful for measuring price change of a single item, being sold for example, from many shops. It has the same form as the wage measurement formula.

$$U_{bi} = \frac{\Sigma_j \; p_{ij} \, q_{ij} \,/\, \Sigma_j \; q_{ij}}{\Sigma_j \; p_{bj} \, q_{bj} \,/\, \Sigma_j \; q_{bj}} \times 100$$

The Normalized Unit Index

As written, the unit index formula can only be used in the case of a single item, being sold for example from various shops. To extend it to the case of several items we have to in a certain sense equate all items in a natural way. One way of doing this is to say that one "normalized unit" of the j-th item, is the amount of that item that can be purchased for one unit of currency based on its mean price $M_j$ over several months (or over a year). The cost of one "normalized unit" of item j in month i is therefore $p_{ij}/M_j$ and the quantity consumed is $q_{ij}M_j$. Thus the value of the normalized unit index in month i now becomes after a little simplification :-

$$N_{bi} = \frac{\Sigma_j \; p_{ij}\, q_{ij} \,/\, \Sigma_j \; q_{ij}M_j}{\Sigma_j \; p_{bj}\, q_{bj} \,/\, \Sigma_j \; q_{bj}M_j} \times 100$$

In words the above formula says the normalized unit index $N_{bi}$ is the change in price expressed as a percentage ratio of one "normalized unit" of each and every
item surveyed where "normalized unit" was defined above.

Regarding defining $M_j$ the mean price of the jth item it is natural to define it as the ratio of the total value of j'th item sold over the desired months to total quantity sold over these months. For example if we only use the data from base and current months we get:-

$$M_j = (p_{ij}\, q_{ij} + p_{bj}\, q_{bj}) \,/\, (q_{ij} + q_{bj})$$

The above formula for $M_j$ is particularly appropriate for chained indices where the previous month is used as the base and we calculate the change in price level from previous to current month. It is also the formula used in the simulation.

If we forget for a moment the interpretation we gave to $M_j$ and allow ourselves to substitute freely on it we can in fact derive both the Paasche and Laspeyres formula from the Normalized Unit Index formula . The substitution $M_j = p_{bj}$ causes that formula to reduce to the Paasche formula. This was pointed out to the author independently by Miriam Tsadiq and by Michael Daly. This caused the author to notice that the substitution $M_j = p_{ij}$ causes that formula to reduce to the Laspeyres formula.

## APPENDIX B - Details of the Simulation

a) We program a hypothetical situation where there are a number of shops selling the same item. The price of this item in month i in shop j is $p_{ij}$ and the quantity sold in month i in shop j is $q_{ij}$ .

b) The main parameters of the simulation of the simulation are:-

elasticity - the elasticity of the relationship between price and quantity
pricevariation - the independent variation (%) of the price level
quantityvariation - the independent variation (%) of the quantity level

c) Independent random number values randomp, randomq are used for causing the independent variation of price and quantity level.

The random numbers "randomp" have mean 1.0 and are uniformly distributed in the range 0 < randomp < 2. They are generated using a linear congruence formula. See the procedure nextrandomp which is given in the next appendix.

Similarly, the random numbers "randomq" have mean 1.0 and are uniformly distributed in the range 0 < randomq < 2. They are generated using a linear congruence formula. See the procedure nextrandomq which is given in the next appendix.

d) The program determines $p_{ij}$ as follows.

$p_{ij}$ := pricemidpoint x ( 1 + (pricevariation / 100.0) x (randomp - 1) );

where pricemidpoint is a constant set to 100.0. Since randomp varies in the range 0 to 2, $p_{ij}$ will vary in the range:

pricemidpoint x ( 1 ± (pricevariation / 100.0) ).

e) The program then determines $q_{ij}$ by firstly determining a quantity midpoint from $p_{ij}$ and the elasticity, and then $q_{ij}$ is determined from this midpoint as follows.

quantitymidpoint := nominalquantity x (($p_{ij}$ / pricemidpoint) ** elasticity );
$q_{ij}$ := quantitymidpoint x ( 1 + (quantityvariation / 100.0) x (randomq-1) );

where nominalquantity is a constant set to 100.0 and ** denotes exponentiation . Similarly since randomq varies in the range 0 to 2, $q_{ij}$ varies in the range:

quantitymidpoint x ( 1 ± (quantityvariation / 100.0)).

f) In each simulation cycle, two sets of price and quantity levels generated ( that is for base month and current month) and one set of index values calculated using the various index formulae. The program then calculates the errors of the various index formulae and outputs these results.

# APPENDIX C - Program Listing

```
PROGRAM index(input,output);

  {WRITTEN BY: R.B.YEHEZKAEL;

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   HAWAAD HALEUMI 21
   JERUSALEM
   ISRAEL.

   DATE: ELUL 5751 - AUGUST 1991 }

CONST
  numberofshops=1000; numberofcycles=1000;
  monthspercycle=2; {BASE MONTH AND CURRENT MONTH}
  evrandoms=1.0; {EXPECTED VALUE OF RANDOM NUMBERS}
  sdrandoms=0.81649658092773; {STANDARD DEVIATION OF THE RANDOM NUMBERS
                               WHICH EQUALS THE SQUARE ROOT OF 2/3}
  programmedindex=100.0; {PROGRAMMED VALUE OF THE INDEX}
  maximumvariation=99; {NOTE - PROBLEMS OF NUMERICAL ACCURACY OCCUR WHEN THE
                        PRICE VARIATION IS 100 AND THE ELASTICITY IS
                        NEGATIVE CAUSED BY DIVISION BY NUMBERS
                        WHICH ARE ALMOST ZERO.  HENCE THIS LIMIT IS NEEDED}
  fulloutputcycles=10; {CYCLES FOR WHICH OUTPUT IS NEEDED IN FULL
                        OUTPUT MODE}
  twotothe15=32768 { 2**15 };  twotothe16=65536 {2**16};

TYPE
  real=double {DOUBLE PRECISION REALS VAX/VMS FEATURE
               GIVING ABOUT 15 SIGNIFICANT DIGITS };

VAR
  seedp,seedq:unsigned; {SEEDS OF RANDOM NUMBER GENERATORS.
                         NOTE unsigned MEANS 32 BIT UNSIGNED INTEGERS
                         THIS IS A VAX/VMS FEATURE}
  randomp,randomq:real; {PRICE RANDOMS AND QUANTITY RANDOMS}
  prandomstoskip,qrandomstoskip:integer; {FOR INITIALIZING RANDOM NUMBERS}
  pricevariation,quantityvariation:real; {THE INDEPENDENT VARIATION IN THE
                                          PRICE LEVEL AND QUANTITY LEVEL}
  percentagestep:integer;
  totalrandomp,totalrandomq:real; {FOR SKEWNESS OF RANDOM NUMBERS}
  randomsusedpercycle, randomsusedinall:real; {THE NUMBER OF PRICE RANDOMS
                                               OR QUANTITY RANDOMS USED}
  cycle,cyclefactor:integer;
  basetotalvalue,basetotalquantity,basetotalnormalizedquantity:real;
  totalvalue,totalquantity,totalnormalizedquantity:real;
  laspeyressum,paaschesum:real;
  laspeyres,paasche,fisher,unitindex,normalizedunitindex:real;
  {THE VALUES OF THE VARIOUS INDICES}
  lerrortotal,perrortotal,ferrortotal,uerrortotal,nerrortotal:real;
  {ERROR TOTALS OF THE LOGARITHMS OF THE ERROR FACTORS (OR RATIOS)}
  summary:ARRAY [1..numberofcycles] OF
  RECORD {SUMMARY INFORMATION ABOUT EACH CYCLE}
    lerror,perror,ferror,uerror,nerror:real;
    {LOGARITHM OF THE ERROR FACTORS (OR RATIOS) OF THE VARIOUS INDICES}
    subtotalrandomp,subtotalrandomq:real;
    {FOR SKEWNESS OF RANDOM NUMBERS}
  END;
  e:real {ELASTICITY};
  p,q:ARRAY [1..numberofshops] OF real
  {CURRENT PRICE AND QUANTITY VECTORS};
  baseprice,basequantity:ARRAY[1..numberofshops]OF real; {BASE PRICES AND QUANTITIES}
  full:boolean; {TRUE FOR FULL LISTING, FALSE FOR SUMMARY LISTING}

  PROCEDURE skipblanks;

  BEGIN
   WHILE (input^ = ' ') AND (NOT eoln) DO get(input);
  END;

  PROCEDURE nextrandomp;

  BEGIN
   {GENERATE RANDOM NUMBERS WITH EXPECTED VALUE 1.0 UNIFORMLY DISTRIBUTED IN
    THE RANGE 0 < randomp < 2 .  THIS ROUTINE IS MACHINE DEPENDENT AND RELIES
```

```
    ON 32 BIT UNSIGNED ARITHMETIC OF VAX/VMS.}

   REPEAT
    seedp:=(seedp*13077+6925); {MOD 2**32}
    {BY USING THE LOWEST 16 BITS OF THE 32 BIT SEED, WE ENSURE
     THAT THE SEQUENCE OF PSEUDO RANDOM NUMBERS GENERATED
     MAY CONTAIN REPETITIONS}
    randomp:=(seedp MOD twotothe16)/twotothe15;
   UNTIL randomp <> 0.0;
  END;

  PROCEDURE nextrandomq;

  BEGIN
   {GENERATE RANDOM NUMBERS WITH EXPECTED VALUE 1.0 UNIFORMLY DISTRIBUTED IN
    THE RANGE 0 < randomq < 2 .  THIS ROUTINE IS MACHINE DEPENDENT AND RELIES
    ON 32 BIT UNSIGNED ARITHMETIC OF VAX/VMS.}

   REPEAT
    seedq:=(seedq*16125+3077); {MOD 2**32}
    {BY USING THE LOWEST 16 BITS OF THE 32 BIT SEED, WE ENSURE
     THAT THE SEQUENCE OF PSEUDO RANDOM NUMBERS GENERATED
     MAY CONTAIN REPETITIONS}
    randomq:=(seedq MOD twotothe16)/twotothe15;
   UNTIL randomq <> 0;
  END;

  PROCEDURE simulatepriceandquantitychanges;

  CONST
    pricemidpoint=100.0; nominalquantity=100.0;
  VAR
    j:integer; {THE SHOP NUMBER}
    quantitymidpoint:real; {THE VALUE OF quantitymidpoint DEPENDS ON p[j] AND e}

  BEGIN
   WITH summary[cycle] DO
     BEGIN

      FOR j:=1 TO numberofshops
       DO
        BEGIN

         {USE RANDOM NUMBER GENERATOR nextrandomp TO VARY PRICES ABOUT pricemidpoint}
         nextrandomp; subtotalrandomp:=subtotalrandomp+randomp;
         p[j]:=pricemidpoint*( 1.0 + (pricevariation/100.0)*(randomp-1.0) );

         {DETERMINE quantitymidpoint FROM THE PRICE p[j] AND THE ELASTICITY e}
         quantitymidpoint:=nominalquantity*((p[j]/pricemidpoint)**e);
         {USE   RANDOM   NUMBER   GENERATOR   nextrandomq   TO   VARY   QUANTITIES   ABOUT
quantitymidpoint}
         nextrandomq; subtotalrandomq:=subtotalrandomq+randomq;
         q[j]:=quantitymidpoint*( 1.0 + (quantityvariation/100.0)*(randomq-1.0)
                               );

        END;

     END;

  END;

  PROCEDURE calculateerrors;

    {CALCULATE ERROR FACTORS (OR RATIOS) OF THE VARIOUS INDICES
     FROM programmedindex AND UPDATE CORRESPONDING ERROR PRODUCTS}

  BEGIN
   WITH summary[cycle] DO
     BEGIN
      lerror:=ln(laspeyres/programmedindex);
      perror:=ln(paasche/programmedindex);
      ferror:=ln(fisher/programmedindex);
      uerror:=ln(unitindex/programmedindex);
      nerror:=ln(normalizedunitindex/programmedindex);
     END;
```

```
END;

PROCEDURE calculateonesetofindices;

VAR
  j:integer; {THE SHOP NUMBER}
  mj:real; {MEAN VALUE OF PRICE IN SHOP NUMBER j BASED ON VALUES
            OF PRICE AND QUANTITY IN BASE AND CURRENT MONTH}

BEGIN

 {CALCULATE SUMS}
 basetotalvalue:=0.0; basetotalquantity:=0.0;
 totalvalue:=0.0; totalquantity:=0.0;
 basetotalnormalizedquantity:=0; totalnormalizedquantity:=0;
 laspeyressum:=0.0; paaschesum:=0.0;
 FOR j:=1 TO numberofshops DO
   BEGIN
    basetotalvalue:=basetotalvalue+baseprice[j]*basequantity[j];
    basetotalquantity:=basetotalquantity+basequantity[j];
    laspeyressum:=laspeyressum+p[j]*basequantity[j];
    paaschesum:=paaschesum+baseprice[j]*q[j];
    totalvalue:=totalvalue+p[j]*q[j];
    totalquantity:=totalquantity+q[j];
    mj:=(baseprice[j]*basequantity[j]+p[j]*q[j]) / (basequantity[j]+q[j]);
    basetotalnormalizedquantity:=basetotalnormalizedquantity + mj*basequantity[j];
    totalnormalizedquantity:=totalnormalizedquantity + mj*q[j];
   END;
 laspeyres:=100.0*laspeyressum/basetotalvalue;
 paasche:=100.0*totalvalue/paaschesum;
 fisher:=sqrt(laspeyres*paasche);
 unitindex:=100.0*(totalvalue/totalquantity) / (basetotalvalue/basetotalquantity);
 normalizedunitindex:=100.0*(totalvalue/totalnormalizedquantity) /
 (basetotalvalue/basetotalnormalizedquantity);

 calculateerrors;

 {FOR FULL LISTING WRITE INDEX VALUES AND ERRORS FOR THE CYCLE}
 IF full AND (cycle <= fulloutputcycles)
  THEN writeln(cycle:4,'  ',
               laspeyres:11:4,paasche:11:4,fisher:11:4,
               unitindex:11:4,normalizedunitindex:11:4);

END;

PROCEDURE simulateonecycle;

VAR
  j:integer; {THE SHOP NUMBER}

BEGIN

 {INITIALIZE SUBTOTALS AND SUMS AND COUNTERS}
 WITH summary[cycle] DO
   BEGIN
    subtotalrandomp:=0.0; subtotalrandomq:=0.0;
   END;

 {GET PRICE AND QUANTITY VALUES FOR BASE MONTH}
 simulatepriceandquantitychanges;

 FOR j:=1 TO numberofshops
  DO BEGIN baseprice[j]:=p[j]; basequantity[j]:=q[j];
     END;

 {GET PRICE AND QUANTITY VALUES FOR CURRENT MONTH}
 simulatepriceandquantitychanges;

 calculateonesetofindices;

END;

PROCEDURE setupfixedvalues;

VAR
  c:char;{FOR SWITCH}
```

```
BEGIN

 {READ FIXED VALUES}
 writeln;

 REPEAT
  writeln('SUMMARY OR FULL LISTING (S/F) ? ');
  skipblanks;
  readln(c);
 UNTIL (c='S') OR (c='s') OR (c='F') OR (c='f');
 full:=(c='F') OR (c='f');

 REPEAT
  writeln('ELASTICITY (2>=...>=-2) ? ');
  readln(e);
 UNTIL (2.0 >= e) AND (e >= -2.0);

 IF full
  THEN
   BEGIN
    REPEAT
     writeln('INDEPENDENT PERCENTAGE VARIATION OF PRICES ( ',
             maximumvariation:1, '>=...>=0 ) ? ');
     readln(pricevariation);
    UNTIL (maximumvariation >= pricevariation) AND (pricevariation >= 0);

    REPEAT
     writeln('INDEPENDENT PERCENTAGE VARIATION OF QUANTITIES ( ',
             maximumvariation:1, '>=...>=0 ) ? ');
     readln(quantityvariation);
    UNTIL (maximumvariation >= quantityvariation) AND (quantityvariation >= 0);
   END
  ELSE
   BEGIN
    REPEAT
     writeln('PERCENTAGE STEP ( ',
             maximumvariation:1, '>=...>=1 ) ? ');
     readln(percentagestep);
    UNTIL (maximumvariation >= percentagestep) AND (percentagestep >= 1);
   END;

 REPEAT
  writeln('NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ? ');
  readln(prandomstoskip);
 UNTIL prandomstoskip >= 0;

 REPEAT
  writeln('NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ? ');
  readln(qrandomstoskip);
 UNTIL qrandomstoskip >= 0;

 {OUTPUT DATA FOR CYCLES WHICH ARE MULTIPLES OF cyclefactor}
 IF full
  THEN cyclefactor:=numberofcycles DIV 10
  ELSE cyclefactor:=numberofcycles; {LAST CYCLE ONLY}

 writeln;
 writeln('SIMULATING ',numberofshops:1,' SHOPS FOR ',
         numberofcycles:1,' CYCLES AND GENERATING ONE SET OF VALUES OF THE');
 writeln('LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH
CYCLE.');

END;

PROCEDURE initializerandomnumbers;

VAR
  n:integer;

BEGIN
 seedp:=0; seedq:=0;
 FOR n:=1 TO prandomstoskip DO nextrandomp;
 FOR n:=1 TO qrandomstoskip DO nextrandomq;
END;

PROCEDURE writesummaries;
```

```
BEGIN

 lerrortotal:=0.0; perrortotal:=0.0; ferrortotal:=0.0;
 uerrortotal:=0.0; nerrortotal:=0.0;
 FOR cycle:=1 TO numberofcycles DO
   WITH summary[cycle] DO
     BEGIN

      {CALCULATE ERROR TOTALS}
      lerrortotal:=lerrortotal+lerror;
      perrortotal:=perrortotal+perror;
      ferrortotal:=ferrortotal+ferror;
      uerrortotal:=uerrortotal+uerror;
      nerrortotal:=nerrortotal+nerror;

      {CALCULATE MEAN OF LOGARITHMIC SUM, GEOMETRIC MEAN OF ERROR FACTORS,
       CONVERT GEOMETRIC MEAN OF ERROR FACTOR TO A PERCENTAGE AND
       WRITE PERCENTAGE ERROR}

      IF full AND ((cycle MOD cyclefactor = 0) OR (cycle <= fulloutputcycles))
       THEN
        BEGIN
         writeln(cycle:4,'  ',
                 100.0*(exp(lerrortotal/cycle)-1.0):11:4,
                 100.0*(exp(perrortotal/cycle)-1.0):11:4,
                 100.0*(exp(ferrortotal/cycle)-1.0):11:4,
                 100.0*(exp(uerrortotal/cycle)-1.0):11:4,
                 100.0*(exp(nerrortotal/cycle)-1.0):11:4);
        END;
      IF (NOT full) AND (cycle MOD cyclefactor = 0)
       THEN
        BEGIN
         writeln(pricevariation:7:1, '   ',
                 quantityvariation:7:1, '       ',
                 100.0*(exp(lerrortotal/cycle)-1.0):11:4,
                 100.0*(exp(perrortotal/cycle)-1.0):11:4,
                 100.0*(exp(ferrortotal/cycle)-1.0):11:4,
                 100.0*(exp(uerrortotal/cycle)-1.0):11:4,
                 100.0*(exp(nerrortotal/cycle)-1.0):11:4);
        END;
     END;

END;

PROCEDURE writerandomnumberstatistics;

BEGIN

 randomsusedpercycle:=monthspercycle*numberofshops;
 randomsusedinall:=0.0;
 totalrandomp:=0.0; totalrandomq:=0.0;
 writeln; writeln;
 IF full
  THEN
   BEGIN {WRITE HEADINGS}
    writeln('THE DEVIATIONS OF THE MEAN OF THE RANDOM NUMBERS, UP TO THE CURRENT
CYCLE,');
    writeln('FROM THEIR EXPECTED VALUE 1.0, IS GIVEN IN THE FOLLOWING TABLE.');
    writeln('THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION');
    writeln('IN PRICE AND QUANTITY LEVELS.');
    writeln;

    writeln('         NUMBER OF STANDARD DEVIATIONS');
    writeln('CYCLE  PRICE RANDOMS     QUANTITY RANDOMS');
    writeln;

   END;
 FOR cycle:=1 TO numberofcycles DO
   WITH summary[cycle] DO
     BEGIN

      randomsusedinall:=randomsusedinall+randomsusedpercycle;
      {CALCULATE TOTALS}
      totalrandomp:=totalrandomp+subtotalrandomp;
      totalrandomq:=totalrandomq+subtotalrandomq;
      IF full AND ((cycle MOD cyclefactor = 0) OR (cycle <= fulloutputcycles))
       THEN writeln(cycle:4,
```

```
                    {MEAN DEVIATIONS OF RANDOM NUMBERS}
                    (totalrandomp/randomsusedinall-evrandoms)/sdrandoms:13:4,
                    '      ',
                    (totalrandomq/randomsusedinall-evrandoms)/sdrandoms:13:4);
     END;

   IF NOT full
    THEN
     BEGIN
      writeln('THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED)');
      writeln('ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE.');
      writeln('THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION');
      writeln('IN PRICE AND QUANTITY LEVELS.  THE DEVIATIONS OF THE MEAN OF THE
UNSCALED');
      writeln('RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE
1.0,');
      writeln('ARE AS FOLLOWS.');
      writeln;
      writeln('PRICE RANDOMS:   ',
              (totalrandomp/randomsusedinall-evrandoms)/sdrandoms:11:4,
              ' STANDARD DEVIATIONS');
      writeln('QUANTITY RANDOMS:',
              (totalrandomq/randomsusedinall-evrandoms)/sdrandoms:11:4,
              ' STANDARD DEVIATIONS');
      writeln;

     END;

  END;

  PROCEDURE simulatenumberofcycles;

  BEGIN

   initializerandomnumbers;

   FOR cycle:=1 TO numberofcycles DO
     BEGIN
      simulateonecycle;
     END;

  END;

  PROCEDURE producefulloutput;

  BEGIN;

   {HEADINGS FOR TABLE OF INDEX VALUES}
   writeln; writeln;
   writeln('       INDEX VALUES FOR CURRENT CYCLE (PROGRAMMED VALUE IS 100)');
   writeln('CYCLE    LASPEYRES   PAASCHE     FISHER     UNIT     NOR.UNIT');
   writeln;

   simulatenumberofcycles;

   {HEADINGS FOR SUMMARY TABLE IN FULL OUTPUT MODE}
   writeln; writeln;
   writeln('THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS
INDICES');
   writeln('WITH RESPECT TO THE INDEX VALUE 100, UP TO THE CURRENT CYCLE.');
   writeln('THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR
FACTORS');
   writeln('WHICH IS CONVERTED TO A PERCENTAGE ERROR.');

   writeln;

   writeln('                  MEAN PERCENTAGE ERROR PER MEASUREMENT');
   writeln('CYCLE    LASPEYRES   PAASCHE     FISHER      UNIT    NOR.UNIT');
   writeln;

   writesummaries;

   writerandomnumberstatistics;

   writeln;
  END;
```

```
  PROCEDURE producesummaryoutput;

  BEGIN
   {HEADINGS FOR SUMMARY TABLE IN SUMMARY OUTPUT MODE}
   writeln; writeln;
   writeln('THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS
INDICES');
   writeln('WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.');
   writeln('THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR
FACTORS');
   writeln('WHICH IS CONVERTED TO A PERCENTAGE ERROR.');

   writeln;

   writeln('INDEPENDENT  VARIATION(%)                    MEAN  PERCENTAGE  ERROR  PER
MEASUREMENT');
   writeln('    PRICE     QUANTITY         LASPEYRES    PAASCHE      FISHER       UNIT
NOR.UNIT');
   {OUTPUT TABLE AT STEP LENGTH}
   pricevariation:=0.0;
   REPEAT
    writeln;
    quantityvariation:=0.0;
    REPEAT
     simulatenumberofcycles;

     writesummaries;

     quantityvariation:=quantityvariation+percentagestep;
    UNTIL quantityvariation > maximumvariation;
    pricevariation:=pricevariation+percentagestep;
   UNTIL pricevariation > maximumvariation;

   writerandomnumberstatistics;

  END;


BEGIN {MAIN PROGRAM}
 setupfixedvalues;

 IF full
  THEN producefulloutput
  ELSE producesummaryoutput;

END.
```

# APPENDIX D - Program Output

```
$ RUN INDEX                    ! FULL OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
   F
ELASTICITY (2>=...>=-2) ?
   -2
INDEPENDENT PERCENTAGE VARIATION OF PRICES ( 99>=...>=0 ) ?
   80
INDEPENDENT PERCENTAGE VARIATION OF QUANTITIES ( 99>=...>=0 ) ?
   80
NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ?
   0
NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ?
   0

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE
LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.
```

INDEX VALUES FOR CURRENT CYCLE (PROGRAMMED VALUE IS 100)

| CYCLE | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
|---|---|---|---|---|---|
| 1 | 210.3599 | 51.2408 | 103.8220 | 106.5939 | 100.7663 |
| 2 | 196.8308 | 50.4352 | 99.6353 | 101.9645 | 98.8895 |
| 3 | 213.5799 | 54.4652 | 107.8549 | 107.7995 | 102.8689 |
| 4 | 202.1844 | 50.8539 | 101.3995 | 106.3425 | 100.8069 |
| 5 | 198.0985 | 50.8661 | 100.3818 | 101.7637 | 100.5987 |
| 6 | 195.7832 | 49.2271 | 98.1725 | 96.7540 | 97.9335 |
| 7 | 206.5940 | 48.4341 | 100.0310 | 100.8903 | 99.4965 |
| 8 | 195.3666 | 50.9008 | 99.7212 | 97.7605 | 100.7009 |
| 9 | 189.5493 | 48.0458 | 95.4309 | 91.2796 | 97.9218 |
| 10 | 203.2589 | 49.7997 | 100.6093 | 101.5776 | 100.4801 |

```
THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES
WITH RESPECT TO THE INDEX VALUE 100, UP TO THE CURRENT CYCLE.
THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS
WHICH IS CONVERTED TO A PERCENTAGE ERROR.
```

MEAN PERCENTAGE ERROR PER MEASUREMENT

| CYCLE | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
|---|---|---|---|---|---|
| 1 | 110.3599 | -48.7592 | 3.8220 | 6.5939 | 0.7663 |
| 2 | 103.4830 | -49.1636 | 1.7071 | 4.2535 | -0.1765 |
| 3 | 106.7944 | -47.9817 | 3.7164 | 5.4224 | 0.8285 |
| 4 | 105.6322 | -48.2753 | 3.1323 | 5.6517 | 0.8231 |
| 5 | 104.1029 | -48.4481 | 2.5762 | 4.8624 | 0.7781 |
| 6 | 102.6921 | -48.8431 | 1.8288 | 3.4653 | 0.2984 |
| 7 | 103.2450 | -49.2412 | 1.5700 | 3.0934 | 0.1834 |
| 8 | 102.2431 | -49.2235 | 1.3370 | 2.4112 | 0.2480 |
| 9 | 100.7917 | -49.5344 | 0.6631 | 1.1102 | -0.0132 |
| 10 | 101.0370 | -49.6014 | 0.6578 | 1.1568 | 0.0360 |
| 100 | 102.5571 | -50.2060 | 0.4297 | 0.5119 | 0.1952 |
| 200 | 102.4146 | -50.2205 | 0.3797 | 0.2711 | 0.1591 |
| 300 | 102.3089 | -50.2826 | 0.2910 | 0.1747 | 0.0975 |
| 400 | 102.1304 | -50.3115 | 0.2176 | 0.0954 | 0.0927 |
| 500 | 102.2238 | -50.2566 | 0.2961 | 0.1549 | 0.1231 |
| 600 | 102.2895 | -50.2725 | 0.2963 | 0.1291 | 0.1007 |
| 700 | 102.2486 | -50.2920 | 0.2665 | 0.1221 | 0.1006 |
| 800 | 102.2059 | -50.2939 | 0.2541 | 0.0907 | 0.0936 |
| 900 | 102.2829 | -50.2800 | 0.2871 | 0.1157 | 0.0916 |
| 1000 | 102.1881 | -50.3198 | 0.2235 | 0.0534 | 0.0724 |

THE DEVIATIONS OF THE MEAN OF THE RANDOM NUMBERS, UP TO THE CURRENT CYCLE, FROM THEIR EXPECTED VALUE 1.0, IS GIVEN IN THE FOLLOWING TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS.

| | NUMBER OF STANDARD DEVIATIONS | |
|---|---|---|
| CYCLE | PRICE RANDOMS | QUANTITY RANDOMS |
| 1 | 0.0239 | -0.0148 |
| 2 | 0.0168 | -0.0172 |
| 3 | 0.0178 | -0.0118 |
| 4 | 0.0158 | -0.0086 |
| 5 | 0.0123 | -0.0090 |
| 6 | 0.0082 | -0.0076 |
| 7 | 0.0053 | -0.0054 |
| 8 | 0.0068 | -0.0047 |
| 9 | 0.0081 | -0.0048 |
| 10 | 0.0093 | -0.0046 |
| 100 | 0.0002 | -0.0002 |
| 200 | 0.0003 | -0.0001 |
| 300 | 0.0002 | -0.0002 |
| 400 | 0.0001 | -0.0001 |
| 500 | 0.0001 | -0.0001 |
| 600 | 0.0002 | -0.0001 |
| 700 | 0.0001 | -0.0001 |
| 800 | 0.0001 | -0.0001 |
| 900 | 0.0001 | -0.0001 |
| 1000 | 0.0001 | -0.0001 |

```
$ RUN INDEX                    ! FULL OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
   F
ELASTICITY (2>=...>=-2) ?
   -2
INDEPENDENT PERCENTAGE VARIATION OF PRICES ( 99>=...>=0 ) ?
   80
INDEPENDENT PERCENTAGE VARIATION OF QUANTITIES ( 99>=...>=0 ) ?
   80
NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ?
   10000
NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ?
   30000
```

**SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.**

**INDEX VALUES FOR CURRENT CYCLE (PROGRAMMED VALUE IS 100)**

| CYCLE | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
|---|---|---|---|---|---|
| 1 | 201.2038 | 50.7410 | 101.0410 | 97.1878 | 99.9621 |
| 2 | 205.3495 | 49.1657 | 100.4796 | 98.3131 | 100.0336 |
| 3 | 199.0162 | 50.1623 | 99.9155 | 100.9485 | 98.8388 |
| 4 | 204.0704 | 48.3997 | 99.3828 | 96.2987 | 97.7060 |
| 5 | 197.9135 | 46.6735 | 96.1110 | 96.4166 | 100.0918 |
| 6 | 190.4018 | 49.1749 | 96.7625 | 97.9502 | 100.1316 |
| 7 | 192.4754 | 49.3403 | 97.4515 | 97.9446 | 100.1693 |
| 8 | 208.4674 | 49.4356 | 101.5170 | 104.0574 | 103.1175 |
| 9 | 201.8735 | 47.4341 | 97.8554 | 98.3684 | 101.8596 |
| 10 | 195.5372 | 44.6470 | 93.4353 | 91.8547 | 98.6310 |

**THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE CURRENT CYCLE. THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.**

**MEAN PERCENTAGE ERROR PER MEASUREMENT**

| CYCLE | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
|---|---|---|---|---|---|
| 1 | 101.2038 | -49.2590 | 1.0410 | -2.8122 | -0.0379 |
| 2 | 103.2661 | -50.0528 | 0.7599 | -2.2511 | -0.0021 |
| 3 | 101.8395 | -49.9812 | 0.4777 | -1.1960 | -0.3900 |
| 4 | 102.3949 | -50.3910 | 0.2028 | -1.8284 | -0.8694 |
| 5 | 101.4906 | -50.9925 | -0.6292 | -2.1819 | -0.6779 |
| 6 | 99.5986 | -50.9647 | -1.0688 | -2.1599 | -0.5435 |
| 7 | 98.5651 | -50.9212 | -1.2815 | -2.1450 | -0.4420 |
| 8 | 99.7767 | -50.8767 | -0.9360 | -1.3904 | -0.0038 |
| 9 | 100.0085 | -51.0674 | -1.0710 | -1.4172 | 0.2015 |
| 10 | 99.5568 | -51.5138 | -1.6346 | -2.1116 | 0.0434 |
| 100 | 102.0746 | -50.5112 | 0.0021 | 0.2340 | 0.0926 |
| 200 | 102.0894 | -50.4962 | 0.0210 | 0.1373 | 0.0473 |
| 300 | 102.0840 | -50.5320 | -0.0166 | 0.0059 | 0.0078 |
| 400 | 101.9494 | -50.5061 | -0.0237 | 0.0498 | 0.0452 |
| 500 | 102.0565 | -50.4856 | 0.0235 | 0.0790 | 0.0564 |
| 600 | 102.1315 | -50.4990 | 0.0286 | 0.0656 | 0.0448 |
| 700 | 102.0633 | -50.5106 | -0.0001 | 0.0481 | 0.0448 |
| 800 | 102.1198 | -50.4936 | 0.0311 | 0.0850 | 0.0431 |
| 900 | 102.1312 | -50.4975 | 0.0299 | 0.0704 | 0.0386 |
| 1000 | 102.0595 | -50.5148 | -0.0052 | 0.0258 | 0.0286 |

THE DEVIATIONS OF THE MEAN OF THE RANDOM NUMBERS, UP TO THE CURRENT CYCLE, FROM THEIR EXPECTED VALUE 1.0, IS GIVEN IN THE FOLLOWING TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS.

| | NUMBER OF STANDARD DEVIATIONS | |
|---|---|---|
| CYCLE | PRICE RANDOMS | QUANTITY RANDOMS |
| 1 | -0.0122 | -0.0205 |
| 2 | -0.0121 | -0.0121 |
| 3 | -0.0023 | -0.0007 |
| 4 | 0.0028 | 0.0047 |
| 5 | 0.0062 | 0.0042 |
| 6 | 0.0057 | 0.0034 |
| 7 | 0.0051 | 0.0027 |
| 8 | 0.0017 | 0.0020 |
| 9 | 0.0020 | 0.0013 |
| 10 | 0.0026 | 0.0016 |
| 100 | -0.0002 | -0.0003 |
| 200 | 0.0000 | 0.0000 |
| 300 | 0.0001 | 0.0001 |
| 400 | 0.0001 | 0.0000 |
| 500 | 0.0000 | 0.0000 |
| 600 | 0.0001 | 0.0000 |
| 700 | 0.0000 | 0.0001 |
| 800 | 0.0000 | 0.0001 |
| 900 | -0.0001 | 0.0001 |
| 1000 | -0.0001 | 0.0001 |

```
$ RUN INDEX                     ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
   S
ELASTICITY (2>=...>=-2) ?
   -2
PERCENTAGE STEP ( 99>=...>=1 ) ?
   20
NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ?
   0
NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ?
   0
```

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.
THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 20.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 40.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 60.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | 2.7615 | -2.6800 | 0.0038 | 0.0044 | 0.0038 |
| 20.0 | 20.0 | 2.7647 | -2.6735 | 0.0086 | 0.0047 | 0.0072 |
| 20.0 | 40.0 | 2.7677 | -2.6669 | 0.0135 | 0.0050 | 0.0109 |
| 20.0 | 60.0 | 2.7707 | -2.6602 | 0.0184 | 0.0052 | 0.0148 |
| 20.0 | 80.0 | 2.7735 | -2.6534 | 0.0233 | 0.0055 | 0.0191 |
| 40.0 | 0.0 | 12.3871 | -11.0060 | 0.0088 | 0.0119 | 0.0081 |
| 40.0 | 20.0 | 12.3975 | -10.9917 | 0.0216 | 0.0124 | 0.0149 |
| 40.0 | 40.0 | 12.4075 | -10.9770 | 0.0342 | 0.0129 | 0.0223 |
| 40.0 | 60.0 | 12.4170 | -10.9620 | 0.0469 | 0.0134 | 0.0305 |
| 40.0 | 80.0 | 12.4260 | -10.9466 | 0.0596 | 0.0139 | 0.0397 |
| 60.0 | 0.0 | 35.1909 | -26.0042 | 0.0177 | 0.0254 | 0.0112 |
| 60.0 | 20.0 | 35.2174 | -25.9814 | 0.0430 | 0.0260 | 0.0224 |
| 60.0 | 40.0 | 35.2423 | -25.9577 | 0.0682 | 0.0265 | 0.0343 |
| 60.0 | 60.0 | 35.2656 | -25.9332 | 0.0934 | 0.0269 | 0.0469 |
| 60.0 | 80.0 | 35.2873 | -25.9080 | 0.1185 | 0.0273 | 0.0606 |
| 80.0 | 0.0 | 101.9879 | -50.4549 | 0.0375 | 0.0573 | 0.0109 |
| 80.0 | 20.0 | 102.0502 | -50.4237 | 0.0845 | 0.0565 | 0.0275 |
| 80.0 | 40.0 | 102.1043 | -50.3908 | 0.1311 | 0.0556 | 0.0436 |
| 80.0 | 60.0 | 102.1502 | -50.3561 | 0.1774 | 0.0545 | 0.0586 |
| 80.0 | 80.0 | 102.1881 | -50.3198 | 0.2235 | 0.0534 | 0.0724 |

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS. THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

```
PRICE RANDOMS:        0.0001 STANDARD DEVIATIONS
QUANTITY RANDOMS:    -0.0001 STANDARD DEVIATIONS
```

```
$ RUN INDEX                    ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
   S
ELASTICITY (2>=...>=-2) ?
   -1
PERCENTAGE STEP ( 99>=...>=1 ) ?
   20
NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ?
   0
NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ?
   0
```

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.
THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 20.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 40.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 60.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | 1.3675 | -1.3420 | 0.0035 | 0.0039 | 0.0036 |
| 20.0 | 20.0 | 1.3699 | -1.3360 | 0.0078 | 0.0041 | 0.0072 |
| 20.0 | 40.0 | 1.3724 | -1.3299 | 0.0121 | 0.0044 | 0.0109 |
| 20.0 | 60.0 | 1.3747 | -1.3238 | 0.0164 | 0.0047 | 0.0149 |
| 20.0 | 80.0 | 1.3770 | -1.3176 | 0.0206 | 0.0050 | 0.0190 |
| 40.0 | 0.0 | 5.9095 | -5.5651 | 0.0078 | 0.0094 | 0.0078 |
| 40.0 | 20.0 | 5.9166 | -5.5527 | 0.0177 | 0.0099 | 0.0148 |
| 40.0 | 40.0 | 5.9235 | -5.5400 | 0.0276 | 0.0105 | 0.0224 |
| 40.0 | 60.0 | 5.9302 | -5.5272 | 0.0376 | 0.0110 | 0.0306 |
| 40.0 | 80.0 | 5.9366 | -5.5142 | 0.0475 | 0.0116 | 0.0394 |
| 60.0 | 0.0 | 15.5090 | -13.4028 | 0.0138 | 0.0181 | 0.0128 |
| 60.0 | 20.0 | 15.5252 | -13.3843 | 0.0315 | 0.0189 | 0.0238 |
| 60.0 | 40.0 | 15.5408 | -13.3653 | 0.0492 | 0.0198 | 0.0356 |
| 60.0 | 60.0 | 15.5558 | -13.3459 | 0.0669 | 0.0206 | 0.0485 |
| 60.0 | 80.0 | 15.5702 | -13.3260 | 0.0846 | 0.0214 | 0.0627 |
| 80.0 | 0.0 | 37.2726 | -27.1147 | 0.0258 | 0.0361 | 0.0183 |
| 80.0 | 20.0 | 37.3099 | -27.0908 | 0.0558 | 0.0370 | 0.0352 |
| 80.0 | 40.0 | 37.3455 | -27.0661 | 0.0857 | 0.0378 | 0.0530 |
| 80.0 | 60.0 | 37.3794 | -27.0405 | 0.1156 | 0.0386 | 0.0717 |
| 80.0 | 80.0 | 37.4116 | -27.0142 | 0.1454 | 0.0394 | 0.0920 |

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS.  THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

```
PRICE RANDOMS:        0.0001 STANDARD DEVIATIONS
QUANTITY RANDOMS:    -0.0001 STANDARD DEVIATIONS
```

```
$ RUN INDEX                    ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
   S
ELASTICITY (2>=...>=-2) ?
   0
PERCENTAGE STEP ( 99>=...>=1 ) ?
   20
NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ?
   0
NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ?
   0
```

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.
THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 20.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 40.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 60.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 |
| 20.0 | 20.0 | 0.0050 | 0.0089 | 0.0070 | 0.0036 | 0.0070 |
| 20.0 | 40.0 | 0.0068 | 0.0145 | 0.0106 | 0.0038 | 0.0109 |
| 20.0 | 60.0 | 0.0085 | 0.0202 | 0.0143 | 0.0041 | 0.0148 |
| 20.0 | 80.0 | 0.0101 | 0.0258 | 0.0180 | 0.0044 | 0.0188 |
| 40.0 | 0.0 | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0065 |
| 40.0 | 20.0 | 0.0101 | 0.0177 | 0.0139 | 0.0071 | 0.0140 |
| 40.0 | 40.0 | 0.0136 | 0.0288 | 0.0212 | 0.0076 | 0.0217 |
| 40.0 | 60.0 | 0.0171 | 0.0401 | 0.0286 | 0.0082 | 0.0295 |
| 40.0 | 80.0 | 0.0205 | 0.0514 | 0.0359 | 0.0088 | 0.0375 |
| 60.0 | 0.0 | 0.0098 | 0.0098 | 0.0098 | 0.0098 | 0.0098 |
| 60.0 | 20.0 | 0.0152 | 0.0264 | 0.0208 | 0.0106 | 0.0210 |
| 60.0 | 40.0 | 0.0205 | 0.0430 | 0.0318 | 0.0114 | 0.0325 |
| 60.0 | 60.0 | 0.0258 | 0.0598 | 0.0428 | 0.0123 | 0.0443 |
| 60.0 | 80.0 | 0.0310 | 0.0766 | 0.0538 | 0.0131 | 0.0561 |
| 80.0 | 0.0 | 0.0130 | 0.0130 | 0.0130 | 0.0130 | 0.0130 |
| 80.0 | 20.0 | 0.0203 | 0.0350 | 0.0276 | 0.0141 | 0.0280 |
| 80.0 | 40.0 | 0.0275 | 0.0571 | 0.0423 | 0.0152 | 0.0433 |
| 80.0 | 60.0 | 0.0346 | 0.0793 | 0.0570 | 0.0163 | 0.0589 |
| 80.0 | 80.0 | 0.0416 | 0.1016 | 0.0716 | 0.0174 | 0.0747 |

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS. THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

```
PRICE RANDOMS:         0.0001 STANDARD DEVIATIONS
QUANTITY RANDOMS:     -0.0001 STANDARD DEVIATIONS
```

```
$ RUN INDEX                     ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
   S
ELASTICITY (2>=...>=-2) ?
   1
PERCENTAGE STEP ( 99>=...>=1 ) ?
   20
NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ?
   0
NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ?
   0

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE
LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.


THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES
WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.
THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS
WHICH IS CONVERTED TO A PERCENTAGE ERROR.
```

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 20.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 40.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 60.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | -1.3107 | 1.3341 | 0.0030 | 0.0027 | 0.0030 |
| 20.0 | 20.0 | -1.3096 | 1.3393 | 0.0061 | 0.0029 | 0.0068 |
| 20.0 | 40.0 | -1.3086 | 1.3446 | 0.0092 | 0.0032 | 0.0107 |
| 20.0 | 60.0 | -1.3075 | 1.3498 | 0.0123 | 0.0035 | 0.0145 |
| 20.0 | 80.0 | -1.3065 | 1.3550 | 0.0154 | 0.0038 | 0.0181 |
| 40.0 | 0.0 | -5.0507 | 5.3303 | 0.0052 | 0.0038 | 0.0051 |
| 40.0 | 20.0 | -5.0503 | 5.3404 | 0.0102 | 0.0043 | 0.0123 |
| 40.0 | 40.0 | -5.0498 | 5.3505 | 0.0153 | 0.0049 | 0.0194 |
| 40.0 | 60.0 | -5.0493 | 5.3606 | 0.0203 | 0.0054 | 0.0263 |
| 40.0 | 80.0 | -5.0487 | 5.3705 | 0.0254 | 0.0059 | 0.0326 |
| 60.0 | 0.0 | -10.6944 | 11.9898 | 0.0065 | 0.0033 | 0.0059 |
| 60.0 | 20.0 | -10.6963 | 12.0050 | 0.0123 | 0.0040 | 0.0150 |
| 60.0 | 40.0 | -10.6981 | 12.0202 | 0.0181 | 0.0047 | 0.0238 |
| 60.0 | 60.0 | -10.6997 | 12.0352 | 0.0239 | 0.0054 | 0.0320 |
| 60.0 | 80.0 | -10.7012 | 12.0501 | 0.0297 | 0.0060 | 0.0393 |
| 80.0 | 0.0 | -17.5580 | 21.3147 | 0.0071 | 0.0012 | 0.0050 |
| 80.0 | 20.0 | -17.5634 | 21.3353 | 0.0124 | 0.0020 | 0.0133 |
| 80.0 | 40.0 | -17.5686 | 21.3559 | 0.0177 | 0.0027 | 0.0211 |
| 80.0 | 60.0 | -17.5738 | 21.3764 | 0.0230 | 0.0035 | 0.0281 |
| 80.0 | 80.0 | -17.5788 | 21.3968 | 0.0283 | 0.0043 | 0.0338 |

```
THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED)
ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE.
THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION
IN PRICE AND QUANTITY LEVELS.  THE DEVIATIONS OF THE MEAN OF THE UNSCALED
RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0,
ARE AS FOLLOWS.

PRICE RANDOMS:        0.0001 STANDARD DEVIATIONS
QUANTITY RANDOMS:    -0.0001 STANDARD DEVIATIONS
```

```
$ RUN INDEX                     ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
   S
ELASTICITY (2>=...>=-2) ?
   2
PERCENTAGE STEP ( 99>=...>=1 ) ?
   20
NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ?
   0
NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ?
   0
```

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.
THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

| INDEPENDENT VARIATION(%) | | MEAN PERCENTAGE ERROR PER MEASUREMENT | | | | |
|---|---|---|---|---|---|---|
| PRICE | QUANTITY | LASPEYRES | PAASCHE | FISHER | UNIT | NOR.UNIT |
| 0.0 | 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 20.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 40.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 60.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0 | 80.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20.0 | 0.0 | -2.5574 | 2.6299 | 0.0026 | 0.0021 | 0.0027 |
| 20.0 | 20.0 | -2.5571 | 2.6348 | 0.0052 | 0.0023 | 0.0065 |
| 20.0 | 40.0 | -2.5567 | 2.6397 | 0.0077 | 0.0026 | 0.0102 |
| 20.0 | 60.0 | -2.5563 | 2.6445 | 0.0103 | 0.0029 | 0.0138 |
| 20.0 | 80.0 | -2.5559 | 2.6493 | 0.0129 | 0.0031 | 0.0171 |
| 40.0 | 0.0 | -9.1794 | 10.1154 | 0.0037 | 0.0016 | 0.0041 |
| 40.0 | 20.0 | -9.1814 | 10.1245 | 0.0068 | 0.0020 | 0.0100 |
| 40.0 | 40.0 | -9.1833 | 10.1335 | 0.0098 | 0.0025 | 0.0157 |
| 40.0 | 60.0 | -9.1850 | 10.1423 | 0.0128 | 0.0029 | 0.0210 |
| 40.0 | 80.0 | -9.1866 | 10.1509 | 0.0159 | 0.0034 | 0.0256 |
| 60.0 | 0.0 | -17.6250 | 21.4049 | 0.0036 | -0.0006 | 0.0045 |
| 60.0 | 20.0 | -17.6304 | 21.4174 | 0.0055 | 0.0000 | 0.0099 |
| 60.0 | 40.0 | -17.6357 | 21.4298 | 0.0074 | 0.0005 | 0.0149 |
| 60.0 | 60.0 | -17.6408 | 21.4420 | 0.0093 | 0.0011 | 0.0192 |
| 60.0 | 80.0 | -17.6457 | 21.4540 | 0.0113 | 0.0016 | 0.0226 |
| 80.0 | 0.0 | -25.9931 | 35.1309 | 0.0031 | -0.0029 | 0.0052 |
| 80.0 | 20.0 | -26.0015 | 35.1453 | 0.0028 | -0.0024 | 0.0090 |
| 80.0 | 40.0 | -26.0097 | 35.1596 | 0.0025 | -0.0018 | 0.0124 |
| 80.0 | 60.0 | -26.0179 | 35.1740 | 0.0023 | -0.0012 | 0.0149 |
| 80.0 | 80.0 | -26.0260 | 35.1883 | 0.0021 | -0.0007 | 0.0161 |

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS. THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

```
PRICE RANDOMS:        0.0001 STANDARD DEVIATIONS
QUANTITY RANDOMS:    -0.0001 STANDARD DEVIATIONS
```