

ב"ה

Interest Free Wage Linkage of Personal Loans and Mortgages

There is a translation into Hebrew:
הצמדת הלוואות ומשכנתאות אישיות לשכר, ללא רבית

Abstract

A case for linkage without interest to the average wage is presented. The accuracy and stability of index formulae is examined and price linkage is compared to wage linkage. The linkage of personal loans and mortgages and relevant Halachic questions are discussed. It is explained how accounting in terms of the average wage can be used as a basis for discouraging fraud and encouraging honesty. The influences of different linkages on economic stability are described.

Author's notes

- 1) Except for the new format, a corrected explanation in section 2, minor corrections, small additions, changes in the order of the material, this version of the paper is identical to the version dated November 1998 - כסלו תשנ"ט. We have made explicit the nature of the controversy regarding the formula for computing a price index and have taken this into account in our conclusions and suggestions.
- 2) The discussion of reciprocal work agreements in the version of this paper dated May 1994 - סיון תשנ"ד - contained an error which has been corrected.
- 3) This work is based on [14] though the discussion of accuracy in section 2 is taken from [11] which treats this issue in detail.

Note of thanks

הודו לה' כי טוב כי לעולם חסדו.

Thanks to the staff and students at Jerusalem College of Technology - Machon Lev, to the rabbis and Torah scholars, to the colleagues, friends and my family, for their encouragement, comments, opposition and criticism. As there are many, I will not single out any one by name, but sincere thanks to everyone.

R.B. Yehezkael (formerly Haskell).

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Foreword

In this paper I will describe a dream which I hope will be realized. This dream is a response to an economic nightmare we experienced in Israel, not so long ago, and from which we must learn.

There were years in which prices increased by 25% every month or two, and penalty interest rates reached about 1760% per annum. I clearly remember seeing a sign on a shop selling car accessories warning - "deferred payments bear interest of 4% a day".

During those years, three zeroes were removed from the currency - remember the lira and the shekel before the new shekel. Sadly, there was a state of economic collapse, and tragically, some committed suicide because of escalating debts.

It is a pity that all this happened amongst a people required by the תורה not to exact interest from each other.

This dream is also a reaction to an old nightmare in which the Jew, and in particular the Jewish moneylender, has been portrayed as a villain, and persecuted. Let us hope, בעזרת השם, that these nightmares never return.

Lending with interest has generated hatred and persecution of the Jewish people and we must not forget the lesson of history. Here I have tried to propose an alternative, which is fair to borrower and lender, and takes into account moral and economic considerations. My prayer and my plea, is that we remember the lesson of history, and make the correct changes.

In spite of all this there were things to be proud of during those years of high inflation. The גמ"ח for example, continued to give interest free loans. Also, I remember a radio program where people phoned in to make contributions, instead of money, some gave "a day", "a week" whilst a friend even contributed "a month". Contributing time, had become more meaningful than contributing money, which was collapsing. These events somehow gave me the idea of interest free linkage to the average wage.

"Time is money" so the saying goes. Money can be lent and borrowed but is this true for time? If time is money, does it have a representative exchange rate (שער יציג)? If time is money, can it be used for defining prices, apartment rentals, fees, balances, monthly payments, rates and so on? By using the average wage for debt linkage, we can get quite close to lending and borrowing time. Yes, time is money and the average wage is its representative exchange rate, its שער יציג. Yes, prices, apartment rentals, fees, balances, monthly payments, rates and so on can be defined in terms of time.

Let me now state that I am not expert and do not have wide knowledge on the topics which I will present. So let me show the gleanings which I have gathered and how they are connected.

This concludes the opening remarks, let us now describe matters in detail.

1 A layman's guide to index formulae

We shall explain by means of a simple example how price and wage indices are calculated. Let us say a family wishes to measure the change in the price of fruit and records details of the "fruit basket" it purchases as follows.

Initial purchase:-

3 kilos apples at 3 coins a kilo and 5 kilos bananas at 2 coins a kilo.

Most recent purchase:-

4 kilos apples at 2 coins a kilo and 1 kilo bananas at 4 coin a kilo.

There are several ways of measuring the change in the family's price index of fruit from the initial to the most recent purchase. We give five methods corresponding to five kinds of indices.

Method 1 - (Laspeyres' Index)

The fruit basket at the initial purchase consisted of 3 kilos apples and 5 kilos bananas. At the time of the initial purchase this basket costs $3 \times 3 + 5 \times 2 = 19$ coins. At the time of the most recent purchase this basket would have cost $3 \times 2 + 5 \times 4 = 26$ coins. The change expressed as a percentage ratio is $26/19 \times 100 = 136.8$. This is the value of Laspeyres' index at the most recent purchase.

Method 2 - (Paasche's Index)

The fruit basket at the most recent purchase consisted of 4 kilos apples and 1 kilo bananas. At the time of the initial purchase this basket would have cost $4 \times 3 + 1 \times 2 = 14$ coins. At the time of the most recent purchase this basket cost $4 \times 2 + 1 \times 4 = 12$ coins. The change expressed as a percentage ratio is $12/14 \times 100 = 85.7$. This is the value of Paasche's index at the most recent purchase.

Method 3 - (Fisher's Index)

Fisher's index is simply the geometric mean (square root of the product) of Laspeyres' and Paasche's indices, that is $\sqrt{(136.8 \times 85.7)} = \sqrt{11723.76} = 108.3$.

Method 4 - (Unit Index)

From the family's viewpoint the cost per kilo of fruit at each purchase is the total amount paid divided by the total kilos bought.

At the initial purchase this is $(3 \times 3 + 5 \times 2) / (3 + 5) = 19/8 = 2.375$ coins per kilo.

At the most recent purchase this is $(4 \times 2 + 1 \times 4) / (4 + 1) = 12/5 = 2.400$ coins per kilo.

The change expressed as a percentage ratio is $2.400/2.375 \times 100 = 101.1$. This is the value of the Unit index at the most recent purchase. (What we have done is calculate the cost per "unit" of fruit, the unit in this case being the kilo). The Unit index is a natural way of measuring change if we restrict ourselves to a single item or similar items which are sold in the same units (see also [11]). However, in the above form it can not be used for measuring price change of different items sold in different units.

Method 5 - (Normalized Unit Index)

To use the unit index in a general situation where we wish to determine the change in price of different items such as food, clothing, furniture, etc. we have to in a certain sense equate all these items in a natural way. One way of doing this is to say that one "normalized unit" of an item, is the amount of that item that can be purchased for one unit of currency based on its mean price for the purchases compared. We now repeat the previous calculation but will treat apples and bananas as different items

The mean price per item over the two purchases is the total amount paid for the items purchased divided by the total amount of items purchased.

For apples this is $(3 \times 3 + 4 \times 2) / (3 + 4) = 17/7$ coins per kilo.

For bananas is $(5 \times 2 + 1 \times 4) / (5 + 1) = 14/6$ coins per kilo.

Therefore the quantity of apples which can be purchased for one coin is $7/17$ kilo and this is the normalized unit for apples.

Similarly the quantity of bananas which can be purchased for one coin is $6/14$ kilo and this is the normalized unit for bananas.

We now calculate for each purchase the total amount paid divided by the total units bought. This gives the cost per normalized unit.

At the initial purchase we have:-

Total cost for 3 kilos apples and 5 kilos bananas is 19 coins (as before).

Total units bought is $3/(7/17) + 5/(6/14) = 3 \times (17/7) + 5 \times (14/6) = 18.952$.

The cost per unit $19/18.952 = 1.003$ coins per normalized unit.

Similarly, at the most recent purchase we have:-

Total cost for 4 kilos apples and 1 kilo bananas is 12 coins (as before).

Total units bought is $4/(7/17) + 1/(6/14) = 4 \times (17/7) + 1 \times (14/6) = 12.048$.

The cost per unit is $12/12.048 = 0.996$ coins per normalized unit.

The change in the cost per unit when expressed as a percentage ratio is $0.996/1.003 \times 100 = 99.3$. This is the value of the Normalized Unit index at the most recent purchase.

Unchained Indices and Chained Indices

In calculating an unchained index, only the data of the initial and most recent purchases are used. A price change ratio is calculated and multiplied by a hundred thereby giving the value of the unchained index.

In calculating a chained index, the data of intermediate purchases are also used. Using these data, price change ratios from purchase to following purchase are determined. The product of these ratios is calculated and multiplied by a hundred thereby giving the value of the chained index.

For example, the chained index for four purchases is calculated as follows. Price change ratios from the first to the second purchase, from the second to the third purchase and from the third to the fourth purchase are determined by the index method chosen. The product of these ratios is calculated and multiplied by a hundred thereby giving the value of the chained index at the fourth purchase.

Notes

- 1) In the previous examples unchained indices were calculated.
- 2) The price change ratio is one hundredth of the unchained index.
- 3) For more information about chaining see [3, 4, 5, 11].

Debt Linkage

Regarding full linkage, the debt and debt repayments would rise and fall in proportion to some index, e.g. prices, wages, foreign currency, etc. For example if index values in three consecutive month were 103, 108, 102, and linked repayments of 50 coins were made each month, and the repayment in the first month was 50 coins, then the repayments in the second month would be $50 \times (108/103) = 52.43$ coins and in the third month this would be $50 \times (102/103) = 49.51$ coins. (In general, the new amount equals the original amount multiplied by the ratio of the new index value to the original index value.)

If the linkage is partial, for example 99% linkage, then the values calculated by using full linkage would be multiplied by 99/100.

Some General Comments

- 1) In Israel, method 1 (Laspeyres' index) is used for prices. That is, we calculate monthly, the change in price of a fixed (or initial) basket of commodities.
- 2) In Israel, method 4 (the Unit index) is used for wages. The unit in this case is the employee post. The cost per employee post is the total wages paid divided by the number of employee posts filled, and this is calculated monthly.
- 3) Methods 2, 3, 5 (Paasche's Fisher's and the Normalized Unit index) are not used generally for price or wage measurement. In this paper they are used in conjunction with the other formulae, to understand the errors and uncertainties in index calculation.
- 4) Here are the values of the index formulae which we calculated.

<u>Index formula</u>	<u>Value</u>
Laspeyres	136.8
Paasche	85.7
Fisher	108.3
Unit	101.1
Normalized Unit	99.3

As can be seen different methods of calculation can give different index values, the greatest difference being between the indices of Laspeyres and Paasche.

2 Accuracy and stability of index formulae

An index formula should be both accurate and stable. "Stable" means that unrealistic prices should have little or no effect on the value of the formula. These matters are particularly important when debts are linked to an index formula.

Accuracy of index formulae

Table 1 (see end of paper) presents a comparison of Laspeyres and Paasche Price indices - see [12]. These data were produced in the United Kingdom and covers the period 1958 to 1967 and gives results for both chained and unchained indices.

Table 2 (see end of paper) presents a comparison of Laspeyres, Paasche, Fisher, Unit and Normalized Unit Indices using computer generated data - see [11]. We simulate a hypothetical situation where there are a number of shops selling the same item. The situation in any month is independent of the situation in other months. Except for random variation, prices and quantities are not changing. Prices, which are the independent variables, are programmed to vary randomly about a fixed midpoint with known random variation. Regarding quantities, five different kinds of relationship between quantity and price are presented in Table 2, namely, (1) quantity inversely proportional to price squared, (2) quantity inversely proportional to price, (3) no correlation between quantity and price, (4) quantity directly proportional to price, (5) quantity directly proportional to price squared.

Apart from random variation, prices and quantities are not changing. Hence on the average, the index should remain 100 which is therefore the true average index value. We use the various index formulae to calculate these indices and compare these calculated values with the pre-programmed value of 100. The mean deviation per measurement from this pre-programmed value is calculated and given in the table. (For a more general treatment and a mathematical explanation see [16], in particular the section "The symmetric case - The true geometric expected index value is known".)

Analysis

The data from Table 1 gives some indication of the errors in index calculation for the U.K., a country and an economy considerably larger than Israel. The data presented here is for the years 1958 to 1967 where inflation was running between 1% to 5% a year (or about 25% for ten years). The results show that for a large economy running at low inflation there is reasonable agreement between the formulae of Laspeyres and Paasche. However whether this agreement can be expected for a small economy running at high inflation is questionable to say the least. Bear in mind Israel experienced inflation of 25% within one or two months not in ten years.

What is disturbing nonetheless in all this is that the difference in the Laspeyres and Paasche indices over ten years (25% inflation) is about 5%. Did we in Israel experience these differences every one or two months and what differences can we expect between these formulae at hundreds of percent of accumulated inflation? To the best of our knowledge a similar comparison has not been done in Israel.

Now, regarding the data from Table 2, we firstly observe that the deviations from the expected value is not significantly changed by the independent quantity variation. We also see that when there is no correlation between quantity and price, all five formulae agree with the pre-programmed index value to within about 0.1% per measurement regardless of the price variation.

On the other hand when quantity is inversely proportional to price (or to price squared), we see very poor accuracy in the formulae of Laspeyres (too high) and Paasche (too low) as the price variation increases. However the Unit index, and the Normalized Unit index agree with the pre-programmed index value to within about 0.1% per measurement regardless of the price variation and Fisher's index to within about 0.2%.

Similarly, when quantity is directly proportional to price (or to price squared), we see very poor accuracy in the formulae of Laspeyres (too low) and Paasche (too high) as the price variation increases. However Fisher's index, the Unit index, and the Normalized Unit index agree with the pre-programmed index value to within about 0.05% per measurement regardless of the price variation.

In addition, since quantity usually increases as price decreases, the cases quantity inversely proportional to price (and to a lesser extent price squared), are more indicative of what is likely to happen in practice. Thus in practice the price index of Laspeyres will be higher and the price index of Paasche lower than the true index

value. This deduction is consistent with the U.K. data in Table 1 which shows Laspeyres' price index always higher than Paasche's price index, but for a more precise explanation see [11] and [4].

In short, we see that the Normalized Unit Index, the Unit Index and Fisher's index were accurate in all our simulations. However, the indices of Laspeyres and Paasche were accurate only when there was no correlation between price and quantity.

Notes

- 1) The issue of accuracy is discussed in more detail in [11]. In [16], it is discussed in a mathematical style.
- 2) As can be seen from table 2, for each value of the independent price variation, there is only a slight change in the values of the mean percentage error per measurement of the various indices, for different values of the independent quantity variation. In other words, these errors depend primarily on the independent price variation which is the independent variable of the simulation.
- 3) It seems preferable to use the Normalized unit index in the chained form, for otherwise there is a danger that after a period of high inflation, the average price per item will be largely determined by most recent prices. There is little chance of this happening when chaining is used, as consecutive months are compared and prices are not that different in consecutive months.
- 4) For the Unit index, the chained and unchained forms are always equal and this can be proved mathematically.

Stability of index formulae

Since Fisher's index is defined as the geometric mean of two inaccurate indices one should be wary of using it. Perhaps in other situations to those we have discussed here, the errors of these two indices will not cancel out and so Fisher's index may be unreliable.

We note that both the Unit index and Normalized unit index have the property of always multiplying price and quantity of an item for the same purchase (section 1). (This is also true for the calculation of normalized units.) This property will give both these indices good stability characteristics since unrealistic prices will typically be down weighted by low quantities. Laspeyres', Paasche's and Fisher's indices do not have this property and therefore may have poor stability characteristics (see also [16]). The following two examples, illustrate this.

Example of a sudden price increase making a price unrealistic.

Initial purchase:-

2 kilos apples at 2 coins a kilo and 2 kilos bananas at 2 coins a kilo.

Most recent purchase:-

4 kilos apples at 2 coins a kilo and 0 kilo bananas at 4 coin a kilo.

Here are the values of the index formulae.

<u>Index formula</u>	<u>Value</u>
Laspeyres	150
Paasche	100
Fisher	122
Unit	100
Normalized Unit	100

Example of a sudden price decrease because of an unrealistic price.

Initial purchase:-

4 kilos apples at 2 coins a kilo and 0 kilos bananas at 4 coins a kilo.

Most recent purchase:-

2 kilos apples at 2 coins a kilo and 2 kilo bananas at 2 coin a kilo.

Here are the values of the index formulae.

<u>Index formula</u>	<u>Value</u>
Laspeyres	100
Paasche	75
Fisher	87
Unit	100
Normalized Unit	100

Note that only the Unit index and Normalized Unit index were not affected by the unrealistic price in both these examples and had value 100 which indicates no change from the buyer's viewpoint.

Finally, the second and third columns in table 4 show the price index and average wage in Israel for the period 1980 to 1986 which included years of high inflation. The price index used is based on Laspeyres' index whereas the average wage means the average wage per employee post. This means that the Unit index for wages in a given month with respect to for example January 1980 will simply be the average wage in that month divided by the average wage in January 1980. So the Unit index with respect January 1980 is in fact proportional to the average wage and has the same percentage rises and falls. We note that during this period, the price index fell once by about 1.3% and that the average wage and therefore also the Unit index for wages fell on eight occasions by a few percent on each occasion. We see in this, evidence of good stability characteristics of the Unit index and poor stability characteristics of Laspeyres' index.

3 Price and wage linkage compared

Table 3 (see end of paper) presents a comparison showing end of year balances and total annual repayments of price and wage linked mortgages. All data in this table is expressed in terms of the average monthly wage. All mortgages in this table are of twenty years duration bearing no interest and with base price index or base average monthly wage being the January values. The start year ranges from 1980 to 1986. The initial repayments for all mortgages whether price linked or wage linked is assumed to be 0.25 of the average monthly wage and the initial debt 60 average monthly wages.

Table 4 (see end of paper) presents a twenty year price linked mortgage bearing no interest with base index January 1980. This table contains monthly repayments and balances expressed in New Shekels and in terms of the average monthly wage. For reference and verification purposes, we also give the values of the price index and the average monthly wage in this table as well as the price/wage ratio. The total annual repayments is given in terms of the average monthly wage only in note 1 to table 4. Similar data was also calculated for the base indices of January (1981 to 1986) and used to produce the comparison table described above.

Analysis

It is evident from the table of the twenty year mortgage that the monthly repayments (when expressed in terms of the average monthly wage) for a price linked mortgage are directly proportional to the price/wage ratio. If we further compare the balances in the comparison table with the aforesaid price/wage ratios, we observe that when the price/wage ratio falls relative to the start of the mortgage, the borrower is better off with price linkage and the lender with wage linkage. If on the other hand this ratio increases relative to the start of the mortgage the borrower is better off with wage linkage and the lender better off with price linkage.

Though we have not produced tables for base months other than January, it appears, that the price/wage ratio is generally lower in the middle of the year than at the year end. So mortgages with base indices in the middle of the year would show a more adverse position for the borrower and a more favourable position for the lender.

In view of these facts we see that wage linkage is not to the detriment of the lender.

In the following sections we shall present stronger arguments in favour of wage linkage.

Other Considerations

There are many formulae for calculating the price index of a number of different items. Fisher for example [15], gives one hundred and thirty four such formulae. He also compared these formulae using data for the U.S.A. covering the years 1913 to 1918. Except for the Unit index formula, he found that there was close agreement between the other four formulae presented here. His findings are different from ours. As the

situations in the U.S.A. and Israel are significantly different from each other, we do not think that his findings are valid for Israel. One thing is certain. There is great controversy regarding the formula for calculating the price index of a number of different items. In view of this, debts should not be linked to such a price index. There is no justification that people should be harassed or have their property confiscated because of debts linked to such a price index, when there is so much controversy regarding its formula.

In the single item case (e.g. wages or price of manpower) there has not been controversy and the Unit index is the accepted choice. In our opinion it is the correct formula in the single item case and as we have seen this index formula has good stability characteristics. These and other reasons indicate that debts may be linked to an index of the average wage or income calculated by the Unit index formula.

Note

The reader interested in comparing our work and Fisher's work [15] should use the following correspondence.

<u>Fisher's formula number</u>	<u>Our formula name</u>
53, 60	Laspeyres
54, 59	Paasche
353	Fisher
52	Unit index
4154	Normalized Unit index

(Fisher does not use a contiguous numbering system for formulae. He also gives several forms for the same formula.)

4 On linkage of personal loans and mortgages

For medium and large debts, the maximum long term linkage that the average wage earner can reasonably handle cannot be higher than the growth of his income. Any higher rate of linkage, will cause a larger and larger portion of the borrower's income to be used for debt repayment with the likelihood of inability to repay the debt. Similarly adding interest to this linkage will also increase the difficulty for the average wage earner to handle debt repayments and the inability of repaying the debt. Wide scale inability of borrowers to repay their debts, can cause the lender not to make a profit or in the worst case go bankrupt because of accumulating bad debts. We therefore see that this form of interest free linkage gives the maximum return which can be reasonably expected from personal loans and mortgages.

Furthermore, full linkage of debts to the average wage would cause hardship to about half the population - namely those whose rate of wage increase is below the average rate of wage increase. In view of this, it is proper to consider partial linkage to the average wage (e.g. 99%), in order that the majority of the population can handle the repayments.

Now regarding the return to be expected from interest free loans linked to the average gross wage, we have calculated as follows. One year fully linked loans would give a return of about 1.5% above the price index to the lender. With 99% linkage, the lender can expect a return of about 0.5% above the price index.

In view of the previous discussion, we suggest that personal loans and mortgages should at most be fully linked to the average wage or income and should bear no interest.

Notes

1) Our observations in section 2 regarding the accuracy of the unit index wage measurement formula and regarding the inaccuracy of the price index formula of Laspeyres should be taken into account here. In view of this, the true return is likely to be higher than calculated above.

2) In doing the above calculations we considered loans which are repayable in one annual payment and have used price and wage data for the years 1980 to 1986 from table 4.

3) Regarding linkage to the average nett wage, corresponding values to the above should be calculated however we do not have the data to do this.

5 Halachic questions

Here we discuss a list of questions where Halachic investigation is needed. However, we do not give definitive answers as this is the province of an Halachic authority.

1) Does debt linkage to an inaccurate index formula contradict laws concerning just weights and measures, laws concerning price fraud, or the biblical requirement "מדבר שקר תרחק..."?

In the book of Genesis we read regarding our forefather Abraham and Ephron the Hittite "And Abraham hearkened to Ephron; and Abraham weighed to Ephron the silver, which he had named in the hearing of the children of Heth, four hundred shekels of silver, current money with the merchant". Thus we see that when making payment, the weighing scale was used as an instrument to measure the value of money, and regarding weights and measures the Torah demands accuracy. It seems to us that these same demands of accuracy similarly apply to the index formulae which are used for debt linkage. Surely accurate formulae must be used whenever debts are linked.

2) Are price linked loans or wage linked loans in keeping of the spirit of the biblical command of "לא תהיה לו... כנ"ש..."? Does price linkage cause hardship to the wage earner?

When prices increase more than wages the borrower pays more with price linkage and less with wage linkage. It therefore seems that wage linkage is in the spirit of the above biblical command, as less is taken from the borrower when he has less at his disposal. On the other hand, price linkage takes more from the borrower when he has less at his disposal and therefore seems to contradict the spirit of the above biblical command and can cause the borrower hardship.

3) Is linkage to the average wage akin to a reciprocal work agreement in which the works of both parties are of equal difficulty and therefore permissible?

Reciprocal work agreements in which one party does harder work than the other party violate interest laws but if the works are of equal difficulty no prohibition is raised, see שולחן ערוך יורה דעה סימן ק"ס סעיף ט' ובבא שולחן ערוך יורה דעה סימן ק"ס סעיף ט' ובבא מציעא פרק ה' משנה י'. So for example if the lender worked for one month in the field for the borrower in summer when it is dry and pleasant, and in return the borrower worked for one month in the field for the lender in winter when it is cold and raining, this would violate interest laws, as work in the field in cold rainy conditions is harder than work in the field in warm pleasant conditions. Furthermore, we see from table 4, that the price/wage ratio is generally lower in summer than in winter, which also indicates that work in summer is generally easier than work in winter. In view of this, there is likely to be objection to lending one average monthly wage in summer and collecting one average wage in winter. However, there seems to be no objection to lending one average monthly wage in any month, and then collecting it in the same month in another year. A much better solution to this difficulty is for example, that the lender gives a loan of one average wage per month over one year and the borrower returns the loan paying one fifth of an average wage per month over five years. This is because one average wage of each of the months of the year is being lent and one average wage of each of the months of the year is being repaid. More generally, the loan can be provided in monthly instalments over one or more full years, and repaid in monthly instalments over one or more full years. This arrangement ensures that payments are made in terms of the average value of different kinds of labour of the whole country over one or more full years. Seasonal and regional fluctuations are thereby neutralized and the risk is lowered.

(However, further investigation is needed to decide whether monthly instalments should be adjusted according to the number of days in a year or according to the number of work days in a year or perhaps no such adjustment is needed at all. For example suppose that adjustment is according to the number of work days and that there are 280 work days in the first year and 270, 290, 275, 285 and 280 work days in the five following years. Monthly instalments for the years following the first year would also be scaled by 280/270, 280/290, 280/275, 280/285 and 280/280 respectively. Perhaps it is possible to avoid this problem by linking to an average hourly wage. See also question 8.)

4) Is linkage to the average wage akin to a partnership?

In a partnership, the partners share in profits and losses. Suppose that the loan is provided in monthly instalments over one or more full years, and repaid in monthly instalments over one or more full years. Suppose that linkage is both to rises and falls of the average wage (section 1). In that case borrower and lender share in the profits and losses caused by rises and falls of the average wage while the loan is being provided and repaid. So here there is similarity. The difference between this arrangement and a partnership

is that the mutual contribution of the parties is not based on the value of their actual labours, but on the average cost of labour.

5) May היתרים for interest be used for lending between Jews?

The following are very different views of two Chief Rabbis of Israel.

"...היתר עיסקא הפך להיות תיקון מקובל ואין לערער עליו...". הרב אברהם שפירא, תחומין כרך ג' תשמ"ב עמ' 240.

"...באשר לריבית, כאן אין מקום לתקנה שתתיר רבית, שכן זו נוגדת את ההלכה, שאין מתנים על רבית. יש אפשרות של הצמדה, כדעת הרשד"ם בניגוד למהרי"ט, וביחוד בתנאי אינפלציה כבימינו, ואפשר ורצוי לתקן שילוו בהצמדה ואז אפילו לא התנה כך, הרי הוא כמו שהתנה. (השווה תוס' ב"מ קד, א, ד"ה "דורשין")...". הרב מרדכי אליהו, תחומין כרך ג' תשמ"ב עמ' 245.

The first view above accepts the use of היתר עיסקא whereas the second view forbids the use of היתרים for interest but permits the linkage of debts without interest. We support the second view and in section 4 suggested that personal loans and mortgages should at most be fully linked to the average wage or income and should bear no interest.

6) Would the lender be getting a fair return on his money with wage linkage?

Recall that in section 3 we presented evidence that wage linkage is not to the detriment of the lender and in section 4 we presented evidence that this form of interest free linkage gives the maximum return which can reasonably be expected from personal loans and mortgages. When wages increase more than prices, the lender's return is increased at the expense of the borrower. So for example, a mortgage bank would receive a timely increase in its income when its wage bill increases (section 7).

7) Are price linked loans near to profit and far from loss and what about wage linked loans?

If we examine the data for the price index and the average monthly wage presented in table 4, we observe that during the period 1980 to 1986 the price index fell in value only once by about 1.3% whereas the average wage fell on eight occasions by about a few percent on each occasion.

8) From an Halachic standpoint, which methods of calculating the average wage or income are acceptable for debt linkage? Which methods are to be preferred?

The unit index is used for measuring wages which seems a good choice. However, the published average monthly wage means the average gross wage per employee post of those employed. Should the nett wage be used? Should those unemployed be included in this average with an income of zero or perhaps other income such as unemployment benefit be taken into account? Is it more correct to use the average hourly wage calculated using the formula: total gross salary paid nationally divided by total hours worked nationally? Is it more correct to use the average hourly wage calculated using the formula: total nett salary received nationally divided by total hours worked nationally? Should average monthly income be used? Should average taxable income be used? Etc. It seems to us that linkage to average nett income is best in agreement with the commandment of "...לא תהיה לו כנִשֶׁה..." and that the unemployed should be included in the calculation, and income from all sources should be taken into account. It also seems to us that linkage to the published average wage is an improvement over linkage to prices, but the unemployed should be included in the calculation. More possibilities appear in [21].

9) From an Halachic standpoint, which formulae for calculating price indices are acceptable for debt linkage? Which formulae are to be preferred?

There are many formulae published in the literature for the measurement of price change of which the formulae of Laspeyres, Paasche, Fisher and the Normalized Unit index are a few. The indices of Laspeyres and Paasche performed poorly on our computational assessment, but the Normalized Unit index and the index of Fisher performed consistently well. However there are other considerations for preferring the Normalized Unit index formula to Fisher's index formula (section 2). In addition, there is great controversy regarding the formula for calculating the price index of a number of different items (section 3). We therefore suggest that debts should not be linked to such a price index.

10) Does the prohibition of "סאה בסאה" see 'סעיף א' have Halachic implications regarding the choice of index formula for debt linkage?

It seems to us that linkage to the formula of Laspeyres or of Paasche violate this prohibition as both these formulae compare the price of a "basket" of commodities which may or may not be available in the market. It also seems to us that debt linkage to Laspeyres' index would be favourable to the lender whereas linkage to Paasche's index is favourable to the borrower. As the index of Fisher is the geometric mean of the index of Laspeyres and Paasche, use of this index for debt linkage may also violate this prohibition. The Unit index formula does not seem to violate this prohibition as the cost per unit is being compared based on what is actually available in the market. Similarly the Normalized unit index does not seem to violate this prohibition and this depends on how "normalized unit" was defined.

11) Which forms of debt linkage are Halachically acceptable when lending to the non-Jew?

Though the Torah permits taking interest from a non-Jew, there was a period in Jewish history where the sages limited who may take interest from a non-Jew or under what conditions this interest may be taken, see 'שולחן ערוך יורה דעה סימן קנ"ט סעיף א'. The *ת"ז* commenting on the *שולחן ערוך* writes that this was done so that Jews should not adopt non-Jewish customs. Also, great care needs to be taken when lending to the non-Jew in view of the considerable suffering Jews have experienced through the caricature of the Jewish moneylender. These considerations are also relevant when lending with linkage to non-Jews and in our opinion the debt linkage policy for such lending, should be morally justifiable and defensible by clear reasoned argument. In our opinion, wage linkage meets these criteria and therefore is to be preferred for such lending. Furthermore, the first point we discussed in this section is equally relevant whether lending to Jew or non-Jew and also needs to be taken into account here.

6 Accounting in terms of the average wage, discouraging fraud, encouraging honesty

Firstly a reservation. In this section we present certain ideas not all of which may be Halachically acceptable. Our aim is to show where to look rather than present a complete answer. We hope that at least some of the ideas will be acceptable but must leave this to the judgment of an Halachic authority. With this in mind we continue.

Theft in Jewish ethical values is a very serious offence. Its prohibition "thou shalt not steal" is given prominence in the eighth commandment. In the book of Proverbs we read "so are the ways of every one greedy of gain; it taketh away the life of the owners thereof". In *Baba Kama* we read "he who steals from his fellow man the value of a Peruta, it is as if he has taken his soul from him". In short, there is a realization in Jewish thought that theft is equated in a certain sense to taking away part of one fellow mans life. Accounting in terms of the average wage makes this fact explicit and quantifiable as we shall see below.

In tables 3, 4 we presented balances and repayments of both price and wage linked mortgages in terms of the average monthly wage. We note that expressing balances and repayments is independent as to how debts are linked or whether they are linked at all. We assert that balances and monies expressed in terms of the average wage gives a clear meaning of the value of these amounts in terms of what it means in time to the average man. Thus we contend that for every average monthly wage stolen, the thief is likely to have "damaged" or "destroyed" one month of life of his fellow man. It seems appropriate therefore that for every average monthly wage stolen, the thief, in the most severe case, should be penalized for a period of one month. This makes explicit the duration of the penalty but what penalty is Halachically acceptable?

In today's society, prison is prescribed as a penalty for theft. The above approach could be used for determining the length of the prison sentence in a meaningful way.

However, the Torah does not prescribe prison as a penalty for theft (or for that matter for any crime). The Torah requires instead the thief return what he has stolen and pay a fine. If the thief is unwilling or unable to do so, he is made to work off the debt. In view of this we suggest that a monthly fine or a higher rate of tax be applied for the time period defined above. If we add the restriction that this penalty ceases the moment the thief pays back what is required, this might make this approach Halachically acceptable.

While making this suggestion it must be made clear that it is not the author's intention that more people should be fined or spend extended periods in prison, but rather to find a framework to deter theft and fraud. Expressing the value of money and the penalty for its theft in like terms, would in our opinion be a significant deterrent.

Theft of lifetime's income

Another possibility which could be considered is to define in law "theft of lifetime's income". This of course requires that average lifetime income be carefully and expertly estimated. (A very approximate guess is about 60 years average income or 720 average monthly wages). We oppose imposing a more severe penalty for such a theft than what we suggested above. Rather, the purpose of making such a definition is to make clear and emphasize in human terms the damage caused by large thefts.

The author further believes it is not enough to deter theft and fraud but it is of the utmost importance to encourage honesty, integrity and impartiality when handling the monies of others. We therefore also suggest that a solemn promise or declaration be made by those responsible for looking after the monies of others and that they are reminded of their promise from time to time. An example of such a promise follows.

An Example of a Solemn Promise of Honesty Integrity and Impartiality

I the undersigned have responsibility for handling and/or deciding on the monies of others and will strive to do so with honesty, integrity and impartiality.

I am aware that if I misuse this responsibility that this will prevent or delay those to whom these monies justly belong from receiving what is due to them. This may cause them hardship, cause them to leave the country, or indeed shorten their lives as the monies involved can be very large and may even exceed the total lifetime income of the average Israeli citizen.

I am aware, G-d forbid, for each average monthly wage stolen or defrauded, I may be "damaging" or "destroying" one month of life of my fellow man.

I am also aware that the maximum penalty for theft and fraud is.....

I therefore will strive to be upright when handling these monies.

7 On linkage and economic stability

It has been suggested by בן-עדן (B. Eden) [13] that linkage to the money supply or to nominal income is less likely to cause inflation than linkage to the price index. In this section we add our arguments as to why linkage to the average wage is less likely to cause inflation than linkage to the price index.

Over the years 1980 to 1985 Israel passed through a period of economic instability and high inflation, during which linkage to a price index was commonly used to fix loan repayments, instalment payments, etc.

Like other countries, Israel uses Laspeyres' formula [1] for calculating price indices. In simple terms the change in the price of a fixed basket of commodities is used to fix the value of the Laspeyres' index. In this context commodity price means the full cash price asked by the seller [1]. Additionally, Laspeyres' index typically overestimates the true index value and in times of high inflation large overestimates may occur (see section 2).

In other words debts have been linked to an index which overestimates full cash asking prices of sellers.

Debt linkage to Laspeyres' price index is prone to cause price instability and escalation. For if prices are raised exceptionally high by sellers, its full cash asking price will drive up the price index and so indirectly increase the government's linked debts. More money will thus be printed enabling these prices to be increased further. Needless to say this cycle repeats itself monthly, as sellers based their prices to a considerable extent on the previous price index value plus a suitable increase, which promptly gets measured by the next price index value. In all this bear in mind:-

1) There is a natural desire to increase full cash asking prices.

2) That the price index statistically overestimates.

The above situation can be described in the terminology of Systems Engineering [7] as a positive feedback loop where an overestimate of price increases is fed back to sellers and used to fix new price and debt levels. It is also well known from the field of Systems Engineering that a positive feedback loop is a cause of instability.

In view of this we assert that though the Laspeyres Price index formula may be suited for giving a rough indication of price change, it is totally unsuited for linkage.

On the other hand the average (monthly) wage [2] is not measured using Laspeyres' formula and does not use the concept of a fixed cross section of wages in its formulation. In simple terms, the change of the total wages paid divided by the number of full time job posts in employment is measured on a month by month basis.

To summarize, we note the following differences between the price index and the average (monthly) wage.

1) The price index is computed using a fixed basket of commodities to estimate consumption levels whereas the average wage is computed using the actual number of full time job posts in employment.

2) The price index measures full cash asking prices whereas the average wage measures actual wages paid.

3) Full cash asking prices may be increased by sellers at their desire whereas actual wages paid are increased after negotiation and agreement of all parties involved.

In view of the above, we expect that an economy which uses the average wage for linkage of debts would behave very differently to an economy where linkage is based on the price index. This difference of behaviour can be better understood by simulating on a computer two basic macroeconomic models in which the relationship between debts, prices, wages and other factors are taken into consideration. In one model debts would be linked to the prices index and in the other model debts would be linked to the average wage. In other respects the models will be identical.

Debt linkage to wages might behave as an automatic stabilizer (negative feedback), since when the standard of living improves, one effectively pays more, and when it decreases one effectively pays less. Thus it gives timely repayment relief to the borrower at the expense of the lender when prices increase more than wages. Therefore wage linkage would have a stabilizing effect on the spending power of borrowers. Furthermore, the return to the lender is increased at the expense of the borrower when wages increase more than prices. So for example, a mortgage bank would receive a timely increase in its income when its wage bill increases.

8 Others who have proposed linkage without interest or zero interest

John von Neumann [17, 18] developed a mathematical model to study economic equilibrium. This means that the proportional structure of the economy is not changing, but its size may change. He found that this can only occur when the interest rate equals the rate of expansion even if these rates are negative. In terms of linkage, this means that debts are linked to the size of the economy without interest. Perhaps John von Neumann is the first to consider interest free linkage even though he did not use this term.

Similarly in the economic model described by Milton Friedman [19], zero nominal interest rates are necessary for efficient resource allocation. Harold L. Cole and Narayana Kocherlakota further developed this approach in [20].

However linkage without interest to the average wage is better suited for personal loans and mortgages, since it takes wages into account directly.

Combined linkages?

It is possible to combine different linkages. For example, the debt can be linked without interest to the size of the economy and be repaid in instalments linked to the average wage. (With such an arrangement, the exact number of instalments may not be known at the time of giving the loan, though it would be known approximately.) Is this combination also suitable for economic equilibrium? Further investigation is needed on this topic.

9 Conclusions and suggestions

We have seen there are several reasons for using linkage to wages and not to prices for loans and other transactions.

We have also seen that interest free wage linkage can give a profit to the lender and is not oppressive to the borrower and so should be used for personal loans and mortgages.

Here are the detailed conclusions followed by the detailed suggestions.

Conclusions

- 1) Full linkage of debts to the average wage or income is the maximum linkage which the average wage earner can reasonably handle (section 4). Adding interest to this linkage will make it very difficult for the average wage earner to handle debt repayments (section 4). This form of interest free linkage gives the maximum return which can be reasonably expected from personal loans and mortgages (section 4).
- 2) Wage linkage is not to the detriment of the lender (section 3). Furthermore, one year interest free loans, fully linked to the average gross wage, would give a return of about 1.5% above the price index (section 4 - based on price and wage data from table 4). However, if we take into account that prices and wages are measured by different index formulae, the true return is likely to be higher (section 4 - note 1).
- 3) Wage linkage is likely to have a more stabilizing effect on the economy than price linkage (section 7). It gives repayment relief to the borrower when prices increase more than wages (section 7). It gives increased income to the lender when wages increase more than prices (section 7).
- 4) Accounting in terms of the average wage can serve as a basis for encouraging the honest handling of monies (section 6).
- 5) Five index formulae are studied in this paper, namely: Laspeyres, Paasche, Fisher, Unit Index, and Normalized Unit Index formulae.
 - a. The Laspeyres and Paasche index formulae had serious inaccuracies in the simulations with high price variation (section 2), and so are likely to give serious errors in times of price instability. In practice we expect Laspeyres' price index to be too high and Paasche's price index to be too low (section 2).
 - b. The Normalized Unit index and Fisher's index formulae gave consistently accurate measurements in all the simulations, but other considerations indicate that the Normalized Unit index formula has good stability characteristics (section 2). So the Normalized Unit index formula is likely to be reliable even in times of price instability. Also, the chained use of this formula is to be preferred (section 2).
 - c. The Unit index formula was consistently accurate in all the simulations and has good stability characteristics (section 2). Its use is appropriate in the case of a single item or similar items sold in the same units (section 1). Thus its use for measuring wages is entirely satisfactory, but it is unsuited for measuring prices.
- 6) There is great controversy regarding the formula for calculating the price index of a number of different items (section 3). In the single item case (e.g. wages) there has not been controversy and the Unit index is the accepted choice (section 3).
- 7) There seem to be Halachic objections to using an inaccurate index formula for debt linkage (section 5). There seem to be Halachic objections to linking debts to a price index which do not apply to linking debts to the average wage or income (section 5).

Suggestions

- 1) Personal loans and mortgages should at most be fully linked to the average wage or income and should bear no interest.
- 2) The various possibilities for wage linkage should be compared, and one of these should be put into practice, e.g. is the linkage to the nett or gross wage, are the unemployed included or excluded in the calculation, and so on (section 5). To ensure that Halachic requirements are met, it seems that the following provisions are needed.
 - a. The loan would need to be provided in monthly instalments over one or more full years, and repaid in monthly instalments over one or more full years (section 5). This arrangement also reduces the risk.
 - b. The loan must be linked to both rises and falls of the average wage (section 5).

- 3) The appropriateness of accounting in terms of the average wage and of enacting laws based thereon to encourage the honest handling of monies needs to be studied by accounting, legal, and Halachic experts.
- 4) Inaccurate index formulae must not be used for debt linkage; accurate formulae must surely be used. In particular, the formulae of Laspeyres and Paasche must not be used for debt linkage. In view of the great controversy regarding the formula for calculating the price index of a number of different items, debts should not be linked to such a price index. In the single item case (e.g. wages) there has not been controversy and the Unit index is the accepted choice. Therefore debts may be linked to an index of the average wage or income calculated by the Unit index formula.
- 5) The Normalized Unit index formula in the chained form should be used to calculate the price index.
- 6) The methods of this paper (section 2 in particular) should be used to analyze how quantity, stock market, and foreign currency indices are calculated.
- 7) Stock exchange, foreign currency, price, and wage indices for the years 1980 to 1986 should be calculated and compared using the five formulae described in this paper. Such a study is likely to give another indication of the inaccuracies in indices in times of high inflation.
- 8) Computer simulations of economies utilizing various wage or price debt linkage policies, using the five formulae described in this paper, should be carried out. This is likely to give an indication of how the method of linkage and the choice of index formula affect economic behaviour, economic stability and inflation.
- 9) Here we have only discussed personal loans and mortgages, but further study is needed to determine if a form of interest free linkage be used for business loans, for example linkage to average business income or gross national product.
- 10) Halachic rulings need to be made on the issues raised in this paper (sections 5, 6 in particular).

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Table 1
Comparison of Laspeyres and Paasche Price Indices for the U.K.

Year	Expenditure per Household	Laspeyres	Paasche	Chain Laspeyres	Chain Paasche
1958	100.0	100.0	100.0	100.0	100.0
1959	104.31987	100.85823	100.54480	100.85823	100.54480
1960	109.14066	101.74241	101.11714	101.72658	101.12728
1961	115.07212	105.03774	103.76592	104.63828	103.85400
1962	117.48085	109.33042	107.86972	108.53627	107.63539
1963	126.63335	111.71473	108.43431	110.67846	109.39957
1964	127.79486	115.58460	112.41367	114.23699	112.62434
1965	143.59308	121.22315	116.13442	120.25972	117.84037
1966	147.08847	126.08628	120.82135	124.86262	122.30600
1967	153.92306	129.44595	123.40484	127.84142	125.42371

Table 2
Comparison of Laspeyres, Paasche, Fisher, Unit
and Normalized Unit Indices using Computer Generated Data

Independent variation(%)		Mean percentage error per measurement				
Price	Quantity	Laspeyres	Paasche	Fisher	Unit	Nor.Unit
<i>Quantity inversely proportional to price squared</i>						
20.0	0.0	2.7615	-2.6800	0.0038	0.0044	0.0038
20.0	80.0	2.7735	-2.6534	0.0233	0.0055	0.0191
40.0	0.0	12.3871	-11.0060	0.0088	0.0119	0.0081
40.0	80.0	12.4260	-10.9466	0.0596	0.0139	0.0397
60.0	0.0	35.1909	-26.0042	0.0177	0.0254	0.0112
60.0	80.0	35.2873	-25.9080	0.1185	0.0273	0.0606
80.0	0.0	101.9879	-50.4549	0.0375	0.0573	0.0109
80.0	80.0	102.1881	-50.3198	0.2235	0.0534	0.0724
<i>Quantity inversely proportional to price</i>						
20.0	0.0	1.3675	-1.3420	0.0035	0.0039	0.0036
20.0	80.0	1.3770	-1.3176	0.0206	0.0050	0.0190
40.0	0.0	5.9095	-5.5651	0.0078	0.0094	0.0078
40.0	80.0	5.9366	-5.5142	0.0475	0.0116	0.0394
60.0	0.0	15.5090	-13.4028	0.0138	0.0181	0.0128
60.0	80.0	15.5702	-13.3260	0.0846	0.0214	0.0627
80.0	0.0	37.2726	-27.1147	0.0258	0.0361	0.0183
80.0	80.0	37.4116	-27.0142	0.1454	0.0394	0.0920
<i>No correlation between quantity and price</i>						
20.0	0.0	0.0033	0.0033	0.0033	0.0033	0.0033
20.0	80.0	0.0101	0.0258	0.0180	0.0044	0.0188
40.0	0.0	0.0065	0.0065	0.0065	0.0065	0.0065
40.0	80.0	0.0205	0.0514	0.0359	0.0088	0.0375
60.0	0.0	0.0098	0.0098	0.0098	0.0098	0.0098
60.0	80.0	0.0310	0.0766	0.0538	0.0131	0.0561
80.0	0.0	0.0130	0.0130	0.0130	0.0130	0.0130
80.0	80.0	0.0416	0.1016	0.0716	0.0174	0.0747
<i>Quantity directly proportional to price</i>						
20.0	0.0	-1.3107	1.3341	0.0030	0.0027	0.0030
20.0	80.0	-1.3065	1.3550	0.0154	0.0038	0.0181
40.0	0.0	-5.0507	5.3303	0.0052	0.0038	0.0051
40.0	80.0	-5.0487	5.3705	0.0254	0.0059	0.0326
60.0	0.0	-10.6944	11.9898	0.0065	0.0033	0.0059
60.0	80.0	-10.7012	12.0501	0.0297	0.0060	0.0393
80.0	0.0	-17.5580	21.3147	0.0071	0.0012	0.0050
80.0	80.0	-17.5788	21.3968	0.0283	0.0043	0.0338
<i>Quantity directly proportional to price squared</i>						
20.0	0.0	-2.5574	2.6299	0.0026	0.0021	0.0027
20.0	80.0	-2.5559	2.6493	0.0129	0.0031	0.0171
40.0	0.0	-9.1794	10.1154	0.0037	0.0016	0.0041
40.0	80.0	-9.1866	10.1509	0.0159	0.0034	0.0256
60.0	0.0	-17.6250	21.4049	0.0036	-0.0006	0.0045
60.0	80.0	-17.6457	21.4540	0.0113	0.0016	0.0226
80.0	0.0	-25.9931	35.1309	0.0031	-0.0029	0.0052
80.0	80.0	-26.0260	35.1883	0.0021	-0.0007	0.0161

Note

The above table is like table 2 in [11] but the results are presented in a different order. To save space, we have not included the entries of table 2 in [11] where the independent price variation is zero. The mean percentage errors are all zero in this case, regardless of the independent quantity variation.

Table 3
Comparison of end of year balances and total annual repayments

The comparison table below gives the year by year position of price linked and wage linked mortgages covering the base (start) months January 1980, January 1981, January 1982, January 1983, January 1984, January 1985, January 1986. All sums are expressed in average monthly wages. With all the mortgages the initial repayment is 0.25 average monthly wages and the initial balance is 60 average monthly wages. The term "End of year balance" means balance after linking to the price index or average wage of December. The base price index and base average monthly wage are for January.

	1980	1981	1982	1983	1984	1985	1986
Base - January 1980							
End of year balance - wage linked	57	54	51	48	45	42	39
End of year Balance - price linked	58.322	50.484	46.294	44.969	36.863	40.848	30.949
Annual repayments - wage linked	3	3	3	3	3	3	3
Annual repayments - price linked	2.913	2.638	2.642	2.501	2.505	2.77	2.551
Base - January 1981							
End of year balance - wage linked		57	54	51	48	45	42
End of year Balance - price linked		56.926	52.363	51.041	42.005	46.753	35.605
Annual repayments - wage linked		3	3	3	3	3	3
Annual repayments - price linked		2.816	2.823	2.674	2.673	2.958	2.726
Base - January 1982							
End of year balance - wage linked			57	54	51	48	45
End of year Balance - price linked			58.968	57.658	47.614	53.205	40.699
Annual repayments - wage linked			3	3	3	3	3
Annual repayments - price linked			3.012	2.853	2.855	3.156	2.91
Base - January 1983							
End of year balance - wage linked				57	54	51	48
End of year Balance - price linked				70.483	58.386	65.468	50.276
Annual repayments - wage linked				3	3	3	3
Annual repayments - price linked				3.305	3.306	3.655	3.37
Base - January 1984							
End of year balance - wage linked					57	54	51
End of year Balance - price linked					51.019	57.385	44.222
Annual repayments - wage linked					3	3	3
Annual repayments - price linked					2.737	3.026	2.788
Base - January 1985							
End of year balance - wage linked						57	54
End of year Balance - price linked						68.876	53.241
Annual repayments - wage linked						3	3
Annual repayments - price linked						3.441	3.17
Base - January 1986							
End of year balance - wage linked							57
End of year Balance - price linked							48.644
Annual repayments - wage linked							3
Annual repayments - price linked							2.745

Table 4
Twenty year mortgage Linked to Price Index but bearing no interest

Original Repayment 0.45525 New Shekel which is equivalent to 0.25 average monthly wages.

Original Debt 109.26 New Shekels which is equivalent to 60 average monthly wages.

Base index - January 1980

Year Months	Price Index Basis 100 in 1976	Avg. Wage New Shekels	Price/ Wage Ratio	Repayments N.S.	Repayments Avg. Wage	Balance N.S.	Balance Avg. Wage
1980							
1	559.9	1.821	307	0.455	0.25	108.805	59.75
2	587.5	1.826	322	0.478	0.262	113.691	62.262
3	617.5	2.144	288	0.502	0.234	118.994	55.501
4	680.5	2.247	303	0.553	0.246	130.581	58.113
5	744.9	2.313	322	0.606	0.262	142.333	61.536
6	779.3	2.575	303	0.634	0.246	148.272	57.581
7	813.4	3.089	263	0.661	0.214	154.099	49.886
8	880.5	3.274	269	0.716	0.219	166.095	50.732
9	944.5	3.208	294	0.768	0.239	177.4	55.299
10	1048.4	3.639	288	0.852	0.234	196.062	53.878
11	1146.7	3.708	309	0.932	0.251	213.513	57.582
12	1215.3	3.863	315	0.988	0.256	225.298	58.322
1981							
1	1304.1138	4.531	288	1.06	0.234	240.703	53.124
2	1375.9152	4.703	293	1.119	0.238	252.836	53.761
3	1441.8723	5.41	267	1.172	0.217	263.784	48.759
4	1595.4939	5.872	272	1.297	0.221	290.591	49.488
5	1648.9275	6.136	269	1.341	0.219	298.983	48.726
6	1694.0121	6.765	250	1.377	0.204	305.78	45.2
7	1796.7048	7.193	250	1.461	0.203	322.856	44.885
8	1866.8364	7.375	253	1.518	0.206	333.94	45.28
9	2018.7882	7.616	265	1.641	0.215	359.48	47.201
10	2200.7964	8.046	274	1.789	0.222	390.1	48.484
11	2328.5361	8.429	276	1.893	0.225	410.849	48.742
12	2448.7617	8.519	287	1.991	0.234	430.071	50.484
1982							
1	2652.4773	9.832	270	2.157	0.219	463.692	47.162
2	2803.5942	10.027	280	2.28	0.227	487.83	48.652
3	2945.5272	11.854	248	2.395	0.202	510.131	43.035
4	3259.4496	12.226	267	2.65	0.217	561.849	45.955
5	3462.3303	12.113	286	2.815	0.232	594.005	49.039
6	3671.0553	13.702	268	2.985	0.218	626.83	45.747
7	4009.1898	15.232	263	3.26	0.214	681.306	44.729
8	4324.782	15.802	274	3.516	0.223	731.42	46.287
9	4652.8977	16.738	278	3.783	0.226	783.129	46.787
10	5043.6309	19.251	262	4.101	0.213	844.792	43.883
11	5370.9117	19.48	276	4.367	0.224	895.244	45.957
12	5668.971	20.312	279	4.609	0.227	940.316	46.294
1983							
1	6149.8734	26.4	233	5.0	0.189	1015.083	38.45
2	6523.0737	25.902	252	5.304	0.205	1071.379	41.363
3	6891.2646	30.233	228	5.603	0.185	1126.249	37.252
4	7805.4801	30.156	259	6.347	0.21	1269.314	42.092
5	8236.2885	30.772	268	6.697	0.218	1332.675	43.308
6	8532.678	34.447	248	6.938	0.201	1373.694	39.878
7	9066.1791	39.357	230	7.372	0.187	1452.212	36.898
8	9715.7313	40.434	240	7.9	0.195	1548.357	38.293
9	10589.0367	40.336	263	8.61	0.213	1678.922	41.623
10	12819.8895	47.522	270	10.424	0.219	2022.206	42.553
11	14765.2065	48.983	301	12.005	0.245	2317.055	47.303
12	16479.2562	57.209	288	13.399	0.234	2572.636	44.969

Table 4 (continued)

Year Months	Price Index Basis 100 in 1976	Avg. Wage New Shekels	Price/ Wage Ratio	Repayments N.S.	Repayments Avg. Wage	Balance N.S.	Balance Avg. Wage
1984							
1	18938.0367	67.3	281	15.398	0.229	2941.087	43.701
2	21204.7902	75.4	281	17.241	0.229	3275.873	43.447
3	23466.5343	89.3	263	19.08	0.214	3606.204	40.383
4	28300.6053	110.0	257	23.011	0.209	4326.065	39.328
5	32346.5307	124.1	261	26.301	0.212	4918.23	39.631
6	36666.3033	155.7	235	29.813	0.191	5545.23	35.615
7	41194.8009	176.0	234	33.495	0.19	6196.602	35.208
8	47980.8681	198.1	242	39.013	0.197	7178.363	36.236
9	58234.275	242.0	241	47.35	0.196	8665.014	35.806
10	72407.5374	277.8	261	58.874	0.212	10715.061	38.571
11	86557.4226	318.5	272	70.379	0.221	12738.62	39.996
12	89791.8252	356.5	252	73.009	0.205	13141.617	36.863
1985							
1	94510.68	381.9	247	76.846	0.201	13755.406	36.018
2	107284.65	395.4	271	87.232	0.221	15527.341	39.27
3	120225.6	511.2	235	97.754	0.191	17302.534	33.847
4	143519.31	530.3	271	116.694	0.22	20538.205	38.729
5	153287.64	583.4	263	124.637	0.214	21811.457	37.387
6	176163.9	685.3	257	143.237	0.209	24923.306	36.368
7	224588.1	743.7	302	182.611	0.246	31591.652	42.479
8	233354.55	769.8	303	189.739	0.246	32635.044	42.394
9	240451.2	749.6	321	195.509	0.261	33432.014	44.6
10	251638.86	777.0	324	204.605	0.263	34782.926	44.766
11	252807.72	806.3	314	205.556	0.255	34738.937	43.084
12	256063.83	856.3	299	208.203	0.243	34978.164	40.848
1986							
1	252661.6125	883.7	286	205.437	0.232	34307.986	38.823
2	256794.3675	902.7	284	208.797	0.231	34660.36	38.396
3	260739.27	1041.5	250	212.005	0.204	34980.811	33.587
4	269380.485	1003.7	268	219.031	0.218	35921.087	35.789
5	273701.0925	979.2	280	222.544	0.227	36274.683	37.045
6	278209.5525	1175.9	237	226.21	0.192	36645.997	31.164
7	278209.5525	1118.2	249	226.21	0.202	36419.787	32.57
8	281215.1925	1041.9	270	228.654	0.219	36584.595	35.113
9	286662.915	1112.3	258	233.083	0.21	37060.231	33.319
10	293425.605	1121.9	262	238.582	0.213	37695.94	33.6
11	301878.9675	1197.9	252	245.455	0.205	38536.476	32.17
12	306387.4275	1255.7	244	249.121	0.198	38862.884	30.949

Notes

1) Repayment totals of the above mortgage expressed in terms of the average wage are as follows.

Year:	1980	1981	1982	1983	1984	1985	1986
Total annual repayments:	2.913	2.638	2.642	2.501	2.505	2.77	2.551

2) Price index taken from publications of the Central Statistical Office and Wage data taken from publications of the National Insurance Institute.

3) In the above the price index represents price level for that month. This value is published on the 15th of the following month.

4) In the above the average monthly gross wage is given. Slightly different (non-provisional) values are published for the average monthly wage for a given month. The general trend is that the later publication has a higher value than the earlier publication. In the above table the later (higher) value is taken.